9.3 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. A sequence is called a ________ sequence if the ratios between consecutive terms are the same. This ratio is called the ________ ratio.
2. The nth term of a geometric sequence has the form ________.
3. The formula for the sum of a finite geometric sequence is given by ________.
4. The sum of the terms of an infinite geometric sequence is called a ________ ________.

SKILLS AND APPLICATIONS

In Exercises 5–16, determine whether the sequence is geometric. If so, find the common ratio.

5. $2, 10, 50, 250, \ldots$
6. $7, 21, 63, 189, \ldots$
7. $3, 12, 21, 30, \ldots$
8. $25, 20, 15, 10, \ldots$
9. $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$
10. $5, 1, 0.2, 0.04, \ldots$
11. $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, \ldots$
12. $9, -6, 4, -\frac{8}{3}, \ldots$
13. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$
14. $\frac{1}{5}, \frac{2}{7}, \frac{3}{9}, \frac{4}{11}, \ldots$
15. $1, -\sqrt{7}, 7, -7\sqrt{7}, \ldots$
16. $2, \frac{4}{\sqrt{3}}, \frac{8}{3}, \frac{16}{3\sqrt{3}}, \ldots$

In Exercises 17–28, write the first five terms of the geometric sequence.

17. $a_1 = 4, r = 3$
18. $a_1 = 8, r = 2$
19. $a_1 = 1, r = \frac{1}{2}$
20. $a_1 = 1, r = \frac{1}{3}$
21. $a_1 = 5, r = -\frac{1}{10}$
22. $a_1 = 6, r = -\frac{1}{4}$
23. $a_1 = 1, r = e$
24. $a_1 = 2, r = \pi$
25. $a_1 = 3, r = \sqrt{5}$
26. $a_1 = 4, r = -\frac{1}{\sqrt{2}}$
27. $a_1 = 2, r = \frac{x}{4}$
28. $a_1 = 5, r = 2x$

In Exercises 29–34, write the first five terms of the geometric sequence. Determine the common ratio and write the nth term of the sequence as a function of n.

29. $a_1 = 64, \quad a_{k+1} = \frac{1}{2}a_k$
30. $a_1 = 81, \quad a_{k+1} = \frac{1}{3}a_k$
31. $a_1 = 9, \quad a_{k+1} = 2a_k$
32. $a_1 = 5, \quad a_{k+1} = -2a_k$
33. $a_1 = 6, \quad a_{k+1} = -\frac{3}{2}a_k$
34. $a_1 = 80, \quad a_{k+1} = -\frac{1}{2}a_k$

In Exercises 35–44, write an expression for the nth term of the geometric sequence. Then find the indicated term.

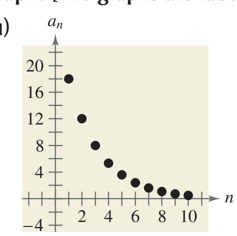
35. $a_1 = 4, r = \frac{1}{2}, n = 10$
36. $a_1 = 5, r = \frac{7}{2}, n = 8$
37. $a_1 = 6, r = -\frac{1}{3}, n = 12$
38. $a_1 = 64, r = -\frac{1}{4}, n = 10$
39. $a_1 = 100, r = e^x, n = 9$
40. $a_1 = 1, r = e^{-x}, n = 4$
41. $a_1 = 1, r = \sqrt{2}, n = 12$
42. $a_1 = 1, r = \sqrt{3}, n = 8$
43. $a_1 = 500, r = 1.02, n = 40$
44. $a_1 = 1000, r = 1.005, n = 60$

In Exercises 45–56, find the indicated nth term of the geometric sequence.

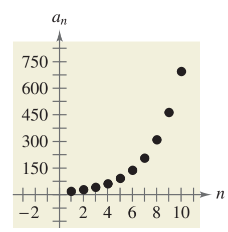
45. 9th term: $11, 33, 99, \ldots$
46. 7th term: $3, 36, 432, \ldots$
47. 10th term: $5, 30, 180, \ldots$
48. 22nd term: $4, 8, 16, \ldots$
49. 8th term: $\frac{1}{2}, -\frac{1}{8}, \frac{1}{32}, -\frac{1}{128}, \ldots$
50. 7th term: $\frac{8}{5}, -\frac{16}{25}, \frac{32}{125}, -\frac{64}{625}, \ldots$
51. 3rd term: $a_1 = 16, a_4 = \frac{27}{4}$
52. 1st term: $a_2 = 3, a_5 = \frac{3}{64}$
53. 6th term: $a_4 = -18, a_7 = \frac{2}{3}$
54. 7th term: $a_3 = \frac{16}{3}, a_5 = \frac{64}{27}$
55. 5th term: $a_2 = 2, a_3 = -\sqrt{2}$
56. 9th term: $a_3 = 11, a_4 = -11\sqrt{11}$

In Exercises 57–60, match the geometric sequence with its graph. [The graphs are labeled (a), (b), (c), and (d).]

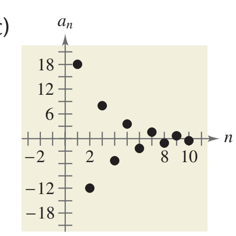
(a)



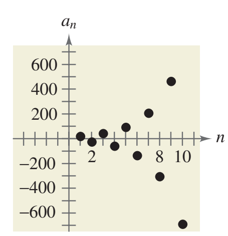
(b)



(c)



(d)



57. $a_n = 18\left(\frac{2}{3}\right)^{n-1}$
58. $a_n = 18\left(-\frac{2}{3}\right)^{n-1}$
59. $a_n = 18\left(\frac{3}{2}\right)^{n-1}$
60. $a_n = 18\left(-\frac{3}{2}\right)^{n-1}$

In Exercises 61–66, use a graphing utility to graph the first 10 terms of the sequence.

61. $a_n = 12(-0.75)^{n-1}$ **62.** $a_n = 10(1.5)^{n-1}$

63. $a_n = 12(-0.4)^{n-1}$ **64.** $a_n = 20(-1.25)^{n-1}$

65. $a_n = 2(1.3)^{n-1}$ **66.** $a_n = 10(1.2)^{n-1}$

In Exercises 67–86, find the sum of the finite geometric sequence.

67. $\sum_{n=1}^{7} 4^{n-1}$ **68.** $\sum_{n=1}^{10} \left(\frac{3}{2}\right)^{n-1}$

69. $\sum_{n=1}^{6} (-7)^{n-1}$ **70.** $\sum_{n=1}^{8} 5\left(-\frac{5}{2}\right)^{n-1}$

71. $\sum_{i=1}^{7} 64\left(-\frac{1}{2}\right)^{i-1}$ **72.** $\sum_{i=1}^{10} 2\left(\frac{1}{4}\right)^{i-1}$

73. $\sum_{i=1}^{6} 32\left(\frac{1}{4}\right)^{i-1}$ **74.** $\sum_{i=1}^{12} 16\left(\frac{1}{2}\right)^{i-1}$

75. $\sum_{n=0}^{20} 3\left(\frac{3}{2}\right)^{n}$ **76.** $\sum_{n=0}^{40} 5\left(\frac{3}{5}\right)^{n}$

77. $\sum_{n=0}^{15} 2\left(\frac{4}{3}\right)^{n}$ **78.** $\sum_{n=0}^{20} 10\left(\frac{1}{5}\right)^{n}$

79. $\sum_{n=0}^{5} 300(1.06)^{n}$ **80.** $\sum_{n=0}^{6} 500(1.04)^{n}$

81. $\sum_{n=0}^{40} 2\left(-\frac{1}{4}\right)^{n}$ **82.** $\sum_{n=0}^{50} 10\left(\frac{2}{3}\right)^{n-1}$

83. $\sum_{i=1}^{10} 8\left(-\frac{1}{4}\right)^{i-1}$ **84.** $\sum_{i=0}^{25} 8\left(-\frac{1}{2}\right)^{i}$

85. $\sum_{i=1}^{10} 5\left(-\frac{1}{3}\right)^{i-1}$ **86.** $\sum_{i=1}^{100} 15\left(\frac{2}{3}\right)^{i-1}$

In Exercises 87–92, use summation notation to write the sum.

87. $10 + 30 + 90 + \cdots + 7290$

88. $9 + 18 + 36 + \cdots + 1152$

89. $2 - \frac{1}{2} + \frac{1}{8} - \cdots + \frac{1}{2048}$

90. $15 - 3 + \frac{3}{5} - \cdots - \frac{3}{625}$

91. $0.1 + 0.4 + 1.6 + \cdots + 102.4$

92. $32 + 24 + 18 + \cdots + 10.125$

In Exercises 93–106, find the sum of the infinite geometric series.

93. $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n}$ **94.** $\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^{n}$

95. $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^{n}$ **96.** $\sum_{n=0}^{\infty} 2\left(-\frac{2}{3}\right)^{n}$

97. $\sum_{n=0}^{\infty} 4\left(\frac{1}{4}\right)^{n}$ **98.** $\sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^{n}$

99. $\sum_{n=0}^{\infty} (0.4)^{n}$ **100.** $\sum_{n=0}^{\infty} 4(0.2)^{n}$

101. $\sum_{n=0}^{\infty} -3(0.9)^{n}$ **102.** $\sum_{n=0}^{\infty} -10(0.2)^{n}$

103. $8 + 6 + \frac{9}{2} + \frac{27}{8} + \cdots$ **104.** $9 + 6 + 4 + \frac{8}{3} + \cdots$

105. $\frac{1}{9} - \frac{1}{3} + 1 - 3 + \cdots$

106. $-\frac{125}{36} + \frac{25}{6} - 5 + 6 - \cdots$

In Exercises 107–110, find the rational number representation of the repeating decimal.

107. $0.\overline{36}$ **108.** $0.\overline{297}$

109. $0.3\overline{18}$ **110.** $1.3\overline{8}$

GRAPHICAL REASONING In Exercises 111 and 112, use a graphing utility to graph the function. Identify the horizontal asymptote of the graph and determine its relationship to the sum.

111. $f(x) = 6\left[\dfrac{1 - (0.5)^x}{1 - (0.5)}\right], \quad \sum_{n=0}^{\infty} 6\left(\dfrac{1}{2}\right)^n$

112. $f(x) = 2\left[\dfrac{1 - (0.8)^x}{1 - (0.8)}\right], \quad \sum_{n=0}^{\infty} 2\left(\dfrac{4}{5}\right)^n$

113. DATA ANALYSIS: POPULATION The table shows the mid-year populations a_n of China (in millions) from 2002 through 2008. (Source: U.S. Census Bureau)

Year	Population, a_n
2002	1284.3
2003	1291.5
2004	1298.8
2005	1306.3
2006	1314.0
2007	1321.9
2008	1330.0

(a) Use the *exponential regression* feature of a graphing utility to find a geometric sequence that models the data. Let n represent the year, with $n = 2$ corresponding to 2002.

(b) Use the sequence from part (a) to describe the rate at which the population of China is growing.

(c) Use the sequence from part (a) to predict the population of China in 2015. The U.S. Census Bureau predicts the population of China will be 1393.4 million in 2015. How does this value compare with your prediction?

(d) Use the sequence from part (a) to determine when the population of China will reach 1.35 billion.

114. COMPOUND INTEREST A principal of \$5000 is invested at 6% interest. Find the amount after 10 years if the interest is compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, and (e) daily.

115. COMPOUND INTEREST A principal of \$2500 is invested at 2% interest. Find the amount after 20 years if the interest is compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, and (e) daily.

116. DEPRECIATION A tool and die company buys a machine for \$175,000 and it depreciates at a rate of 30% per year. (In other words, at the end of each year the depreciated value is 70% of what it was at the beginning of the year.) Find the depreciated value of the machine after 5 full years.

117. ANNUITIES A deposit of \$100 is made at the beginning of each month in an account that pays 6% interest, compounded monthly. The balance A in the account at the end of 5 years is

$$A = 100\left(1 + \frac{0.06}{12}\right)^1 + \cdot \cdot \cdot + 100\left(1 + \frac{0.06}{12}\right)^{60}.$$

Find A.

118. ANNUITIES A deposit of \$50 is made at the beginning of each month in an account that pays 8% interest, compounded monthly. The balance A in the account at the end of 5 years is

$$A = 50\left(1 + \frac{0.08}{12}\right)^1 + \cdot \cdot \cdot + 50\left(1 + \frac{0.08}{12}\right)^{60}.$$

Find A.

119. ANNUITIES A deposit of P dollars is made at the beginning of each month in an account with an annual interest rate r, compounded monthly. The balance A after t years is

$$A = P\left(1 + \frac{r}{12}\right) + P\left(1 + \frac{r}{12}\right)^2 + \cdot \cdot \cdot + P\left(1 + \frac{r}{12}\right)^{12t}.$$

Show that the balance is

$$A = P\left[\left(1 + \frac{r}{12}\right)^{12t} - 1\right]\left(1 + \frac{12}{r}\right).$$

120. ANNUITIES A deposit of P dollars is made at the beginning of each month in an account with an annual interest rate r, compounded continuously. The balance A after t years is

$$A = Pe^{r/12} + Pe^{2r/12} + \cdot \cdot \cdot + Pe^{12tr/12}.$$

Show that the balance is $A = \dfrac{Pe^{r/12}(e^{rt} - 1)}{e^{r/12} - 1}$.

ANNUITIES In Exercises 121–124, consider making monthly deposits of P dollars in a savings account with an annual interest rate r. Use the results of Exercises 119 and 120 to find the balance A after t years if the interest is compounded (a) monthly and (b) continuously.

121. $P = \$50,\ r = 5\%,\ t = 20$ years

122. $P = \$75,\ r = 3\%,\ t = 25$ years

123. $P = \$100,\ r = 2\%,\ t = 40$ years

124. $P = \$20,\ r = 4.5\%,\ t = 50$ years

125. ANNUITIES Consider an initial deposit of P dollars in an account with an annual interest rate r, compounded monthly. At the end of each month, a withdrawal of W dollars will occur and the account will be depleted in t years. The amount of the initial deposit required is

$$P = W\left(1 + \frac{r}{12}\right)^{-1} + W\left(1 + \frac{r}{12}\right)^{-2} + \cdot \cdot \cdot + W\left(1 + \frac{r}{12}\right)^{-12t}.$$

Show that the initial deposit is

$$P = W\left(\frac{12}{r}\right)\left[1 - \left(1 + \frac{r}{12}\right)^{-12t}\right].$$

126. ANNUITIES Determine the amount required in a retirement account for an individual who retires at age 65 and wants an income of \$2000 from the account each month for 20 years. Use the result of Exercise 125 and assume that the account earns 9% compounded monthly.

MULTIPLIER EFFECT In Exercises 127–130, use the following information. A tax rebate has been given to property owners by the state government with the anticipation that each property owner will spend approximately p% of the rebate, and in turn each recipient of this amount will spend p% of what they receive, and so on. Economists refer to this exchange of money and its circulation within the economy as the "multiplier effect." The multiplier effect operates on the idea that the expenditures of one individual become the income of another individual. For the given tax rebate, find the total amount put back into the state's economy, if this effect continues without end.

	Tax rebate	*p%*
127.	\$400	75%
128.	\$250	80%
129.	\$600	72.5%
130.	\$450	77.5%