9.2   EXERCISES

VOCABULARY: Fill in the blanks.

1. A sequence is called an _____ sequence if the differences between consecutive terms are the same.
   This difference is called the _____ difference.

2. The nth term of an arithmetic sequence has the form _____.

3. If you know the nth term of an arithmetic sequence *and* you know the common difference of the sequence,
   you can find the $(n + 1)$th term by using the _____ formula $a_{n+1} = a_n + d$.

4. The formula $S_n = \dfrac{n}{2}(a_1 + a_n)$ can be used to find the sum of the first n terms of an arithmetic sequence,

   called the _____ of a _____ _____ _____.

SKILLS AND APPLICATIONS

In Exercises 5–14, determine whether the sequence is arithmetic. If so, find the common difference.

5. 10, 8, 6, 4, 2, . . .   **6.** 4, 9, 14, 19, 24, . . .

7. 1, 2, 4, 8, 16, . . .   **8.** 80, 40, 20, 10, 5, . . .

9. $\frac{9}{4}, 2, \frac{7}{4}, \frac{3}{2}, \frac{5}{4}, . . .$ **10.** $3, \frac{5}{2}, 2, \frac{3}{2}, 1, . . .$

11. 3.7, 4.3, 4.9, 5.5, 6.1, . . .

12. 5.3, 5.7, 6.1, 6.5, 6.9, . . .

13. ln 1, ln 2, ln 3, ln 4, ln 5, . . .

14. $1^2, 2^2, 3^2, 4^2, 5^2, . . .$

In Exercises 15–22, write the first five terms of the sequence. Determine whether the sequence is arithmetic. If so, find the common difference. (Assume that n begins with 1.)

15. $a_n = 5 + 3n$ **16.** $a_n = 100 - 3n$

17. $a_n = 3 - 4(n - 2)$ **18.** $a_n = 1 + (n - 1)4$

19. $a_n = (-1)^n$ **20.** $a_n = 2^{n-1}$

21. $a_n = \dfrac{(-1)^n 3}{n}$ **22.** $a_n = (2^n)n$

In Exercises 23–32, find a formula for a_n for the arithmetic sequence.

23. $a_1 = 1, d = 3$ **24.** $a_1 = 15, d = 4$

25. $a_1 = 100, d = -8$ **26.** $a_1 = 0, d = -\frac{2}{3}$

27. $4, \frac{3}{2}, -1, -\frac{7}{2}, . . .$ **28.** $10, 5, 0, -5, -10, . . .$

29. $a_1 = 5, a_4 = 15$ **30.** $a_1 = -4, a_5 = 16$

31. $a_3 = 94, a_6 = 85$ **32.** $a_5 = 190, a_{10} = 115$

In Exercises 33–40, write the first five terms of the arithmetic sequence.

33. $a_1 = 5, d = 6$

34. $a_1 = 5, d = -\frac{3}{4}$

35. $a_1 = -2.6, d = -0.4$

36. $a_1 = 16.5, d = 0.25$

37. $a_1 = 2, a_{12} = 46$

38. $a_4 = 16, a_{10} = 46$

39. $a_8 = 26, a_{12} = 42$

40. $a_3 = 19, a_{15} = -1.7$

In Exercises 41–46, write the first five terms of the arithmetic sequence defined recursively.

41. $a_1 = 15, \quad a_{n+1} = a_n + 4$

42. $a_1 = 6, \quad a_{n+1} = a_n + 5$

43. $a_1 = 200, \quad a_{n+1} = a_n - 10$

44. $a_1 = 72, \quad a_{n+1} = a_n - 6$

45. $a_1 = \frac{5}{8}, \quad a_{n+1} = a_n - \frac{1}{8}$

46. $a_1 = 0.375, \quad a_{n+1} = a_n + 0.25$

In Exercises 47–50, the first two terms of the arithmetic sequence are given. Find the missing term.

47. $a_1 = 5, a_2 = 11, a_{10} = $ �

48. $a_1 = 3, a_2 = 13, a_9 = $

49. $a_1 = 4.2, a_2 = 6.6, a_7 = $

50. $a_1 = -0.7, a_2 = -13.8, a_8 = $

In Exercises 51–58, find the sum of the finite arithmetic sequence.

51. $2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20$

52. $1 + 4 + 7 + 10 + 13 + 16 + 19$

53. $-1 + (-3) + (-5) + (-7) + (-9)$

54. $-5 + (-3) + (-1) + 1 + 3 + 5$

55. Sum of the first 50 positive even integers

56. Sum of the first 100 positive odd integers

57. Sum of the integers from -100 to 30

58. Sum of the integers from -10 to 50

In Exercises 59–66, find the indicated nth partial sum of the arithmetic sequence.

59. $8, 20, 32, 44, \ldots, \quad n = 10$

60. $-6, -2, 2, 6, \ldots, \quad n = 50$

61. $4.2, 3.7, 3.2, 2.7, \ldots, \quad n = 12$

62. $0.5, 1.3, 2.1, 2.9, \ldots, \quad n = 10$

63. $40, 37, 34, 31, \ldots, \quad n = 10$

64. $75, 70, 65, 60, \ldots, \quad n = 25$

65. $a_1 = 100, \ a_{25} = 220, \quad n = 25$

66. $a_1 = 15, \ a_{100} = 307, \quad n = 100$

In Exercises 67–74, find the partial sum.

67. $\displaystyle\sum_{n=1}^{50} n$

68. $\displaystyle\sum_{n=1}^{100} 2n$

69. $\displaystyle\sum_{n=10}^{100} 6n$

70. $\displaystyle\sum_{n=51}^{100} 7n$

71. $\displaystyle\sum_{n=11}^{30} n - \sum_{n=1}^{10} n$

72. $\displaystyle\sum_{n=51}^{100} n - \sum_{n=1}^{50} n$

73. $\displaystyle\sum_{n=1}^{500} (n + 8)$

74. $\displaystyle\sum_{n=1}^{250} (1000 - n)$

In Exercises 75–78, match the arithmetic sequence with its graph. [The graphs are labeled (a), (b), (c), and (d).]

(a)

(b)

(c)

(d)
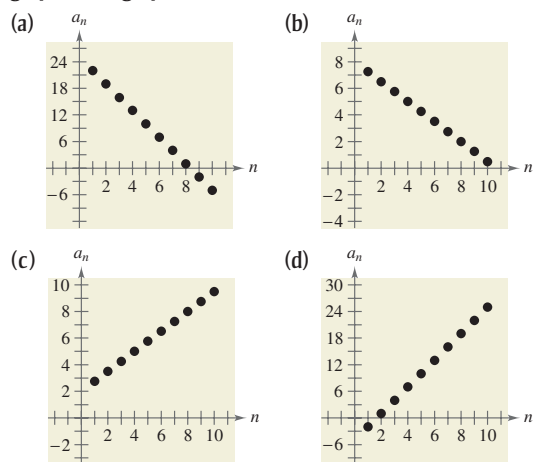
75. $a_n = -\frac{3}{4}n + 8$

76. $a_n = 3n - 5$

77. $a_n = 2 + \frac{3}{4}n$

78. $a_n = 25 - 3n$

In Exercises 79–82, use a graphing utility to graph the first 10 terms of the sequence. (Assume that n begins with 1.)

79. $a_n = 15 - \frac{3}{2}n$

80. $a_n = -5 + 2n$

81. $a_n = 0.2n + 3$

82. $a_n = -0.3n + 8$

In Exercises 83–88, use a graphing utility to find the partial sum.

83. $\displaystyle\sum_{n=1}^{20} (2n + 1)$

84. $\displaystyle\sum_{n=0}^{50} (50 - 2n)$

85. $\displaystyle\sum_{n=1}^{100} \frac{n + 1}{2}$

86. $\displaystyle\sum_{n=0}^{100} \frac{4 - n}{4}$

87. $\displaystyle\sum_{i=1}^{60} \left(250 - \frac{2}{5}i\right)$

88. $\displaystyle\sum_{j=1}^{200} (10.5 + 0.025j)$

JOB OFFER In Exercises 89 and 90, consider a job offer with the given starting salary and the given annual raise.

(a) Determine the salary during the sixth year of employment.

(b) Determine the total compensation from the company through six full years of employment.

Starting Salary	Annual Raise
89. $32,500	$1500
90. $36,800	$1750

91. SEATING CAPACITY Determine the seating capacity of an auditorium with 30 rows of seats if there are 20 seats in the first row, 24 seats in the second row, 28 seats in the third row, and so on.

92. SEATING CAPACITY Determine the seating capacity of an auditorium with 36 rows of seats if there are 15 seats in the first row, 18 seats in the second row, 21 seats in the third row, and so on.

93. BRICK PATTERN A brick patio has the approximate shape of a trapezoid (see figure). The patio has 18 rows of bricks. The first row has 14 bricks and the 18th row has 31 bricks. How many bricks are in the patio?
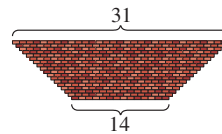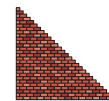

FIGURE FOR 93

FIGURE FOR 94

94. BRICK PATTERN   A triangular brick wall is made by cutting some bricks in half to use in the first column of every other row (see figure on the previous page). The wall has 28 rows. The top row is one-half brick wide and the bottom row is 14 bricks wide. How many bricks are used in the finished wall?

95. FALLING OBJECT   An object with negligible air resistance is dropped from a plane. During the first second of fall, the object falls 4.9 meters; during the second second, it falls 14.7 meters; during the third second, it falls 24.5 meters; during the fourth second, it falls 34.3 meters. If this arithmetic pattern continues, how many meters will the object fall in 10 seconds?

96. FALLING OBJECT   An object with negligible air resistance is dropped from the top of the Sears Tower in Chicago at a height of 1454 feet. During the first second of fall, the object falls 16 feet; during the second second, it falls 48 feet; during the third second, it falls 80 feet; during the fourth second, it falls 112 feet. If this arithmetic pattern continues, how many feet will the object fall in 7 seconds?

97. PRIZE MONEY   A county fair is holding a baked goods competition in which the top eight bakers receive cash prizes. First place receives a cash prize of $200, second place receives $175, third place receives $150, and so on.

(a) Write a sequence a_n that represents the cash prize awarded in terms of the place n in which the baked good places.

(b) Find the total amount of prize money awarded at the competition.

98. PRIZE MONEY   A city bowling league is holding a tournament in which the top 12 bowlers with the highest three-game totals are awarded cash prizes. First place will win $1200, second place $1100, third place $1000, and so on.

(a) Write a sequence a_n that represents the cash prize awarded in terms of the place n in which the bowler finishes.

(b) Find the total amount of prize money awarded at the tournament.

99. TOTAL PROFIT   A small snowplowing company makes a profit of $8000 during its first year. The owner of the company sets a goal of increasing profit by $1500 each year for 5 years. Assuming that this goal is met, find the total profit during the first 6 years of this business. What kinds of economic factors could prevent the company from meeting its profit goal? Are there any other factors that could prevent the company from meeting its goal? Explain.

100. TOTAL SALES   An entrepreneur sells $15,000 worth of sports memorabilia during one year and sets a goal of increasing annual sales by $5000 each year for 9 years. Assuming that this goal is met, find the total sales during the first 10 years of this business. What kinds of economic factors could prevent the business from meeting its goals?

101. BORROWING MONEY   You borrowed $2000 from a friend to purchase a new laptop computer and have agreed to pay back the loan with monthly payments of $200 plus 1% interest on the unpaid balance.

(a) Find the first six monthly payments you will make, and the unpaid balance after each month.

(b) Find the total amount of interest paid over the term of the loan.

102. BORROWING MONEY   You borrowed $5000 from your parents to purchase a used car. The arrangements of the loan are such that you will make payments of $250 per month plus 1% interest on the unpaid balance.

(a) Find the first year's monthly payments you will make, and the unpaid balance after each month.

(b) Find the total amount of interest paid over the term of the loan.

103. DATA ANALYSIS: PERSONAL INCOME   The table shows the per capita personal income a_n in the United States from 2002 through 2008. (Source: U.S. Bureau of Economic Analysis)

Year	Per capita personal income, a_n
2002	$30,834
2003	$31,519
2004	$33,159
2005	$34,691
2006	$36,791
2007	$38,654
2008	$39,742

(a) Find an arithmetic sequence that models the data. Let n represent the year, with $n = 2$ corresponding to 2002.

(b) Use a graphing utility to graph the terms of the finite sequence you found in part (a).

(c) Use the sequence from part (a) to estimate the per capita personal income in 2009.

(d) Use your school's library, the Internet, or some other reference source to find the actual per capita personal income in 2009, and compare this value with the estimate from part (c).