9.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. An ________ ________ is a function whose domain is the set of positive integers.

2. The function values $a_1, a_2, a_3, a_4, \ldots$ are called the ________ of a sequence.

3. A sequence is a ________ sequence if the domain of the function consists only of the first n positive integers.

4. If you are given one or more of the first few terms of a sequence, and all other terms of the sequence are defined using previous terms, then the sequence is said to be defined ________.

5. If n is a positive integer, n ________ is defined as $n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots (n-1) \cdot n$.

6. The notation used to represent the sum of the terms of a finite sequence is ________ ________ or sigma notation.

7. For the sum $\sum_{i=1}^{n} a_i$, i is called the ________ of summation, n is the ________ limit of summation, and 1 is the ________ limit of summation.

8. The sum of the terms of a finite or infinite sequence is called a ________.

SKILLS AND APPLICATIONS

In Exercises 9–32, write the first five terms of the sequence. (Assume that n begins with 1.)

9. $a_n = 2n + 5$

10. $a_n = 4n - 7$

11. $a_n = 2^n$

12. $a_n = \left(\frac{1}{2}\right)^n$

13. $a_n = (-2)^n$

14. $a_n = \left(-\frac{1}{2}\right)^n$

15. $a_n = \dfrac{n+2}{n}$

16. $a_n = \dfrac{n}{n+2}$

17. $a_n = \dfrac{6n}{3n^2 - 1}$

18. $a_n = \dfrac{2n}{n^2 + 1}$

19. $a_n = \dfrac{1 + (-1)^n}{n}$

20. $a_n = 1 + (-1)^n$

21. $a_n = 2 - \dfrac{1}{3^n}$

22. $a_n = \dfrac{2^n}{3^n}$

23. $a_n = \dfrac{1}{n^{3/2}}$

24. $a_n = \dfrac{10}{n^{2/3}}$

25. $a_n = \dfrac{(-1)^n}{n^2}$

26. $a_n = (-1)^n\left(\dfrac{n}{n+1}\right)$

27. $a_n = \frac{2}{3}$

28. $a_n = 0.3$

29. $a_n = n(n-1)(n-2)$

30. $a_n = n(n^2 - 6)$

31. $a_n = \dfrac{(-1)^{n+1}}{n^2 + 1}$

32. $a_n = \dfrac{(-1)^{n+1}}{2n + 1}$

In Exercises 33–36, find the indicated term of the sequence.

33. $a_n = (-1)^n(3n - 2)$

$a_{25} =$ ▢

34. $a_n = (-1)^{n-1}[n(n-1)]$

$a_{16} =$ ▢

35. $a_n = \dfrac{4n}{2n^2 - 3}$

$a_{11} =$ ▢

36. $a_n = \dfrac{4n^2 - n + 3}{n(n-1)(n+2)}$

$a_{13} =$ ▢

In Exercises 37–42, use a graphing utility to graph the first 10 terms of the sequence. (Assume that n begins with 1.)

37. $a_n = \dfrac{2}{3}n$

38. $a_n = 2 - \dfrac{4}{n}$

39. $a_n = 16(-0.5)^{n-1}$

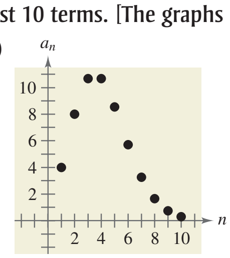
40. $a_n = 8(0.75)^{n-1}$

41. $a_n = \dfrac{2n}{n+1}$

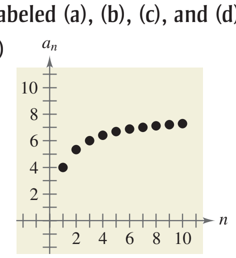
42. $a_n = \dfrac{3n^2}{n^2 + 1}$

In Exercises 43–46, match the sequence with the graph of its first 10 terms. [The graphs are labeled (a), (b), (c), and (d).]

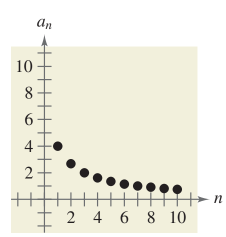
(a)



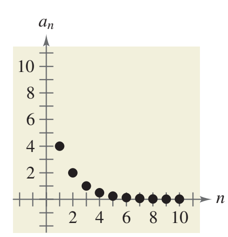
(b)



(c)



(d)



43. $a_n = \dfrac{8}{n+1}$

44. $a_n = \dfrac{8n}{n+1}$

45. $a_n = 4(0.5)^{n-1}$

46. $a_n = \dfrac{4^n}{n!}$

In Exercises 47–62, write an expression for the apparent nth term of the sequence. (Assume that n begins with 1.)

47. $1, 4, 7, 10, 13, \ldots$

48. $3, 7, 11, 15, 19, \ldots$

49. $0, 3, 8, 15, 24, \ldots$

50. $2, -4, 6, -8, 10, \ldots$

51. $-\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \frac{5}{6}, -\frac{6}{7}, \ldots$

52. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \ldots$

53. $\frac{2}{1}, \frac{3}{3}, \frac{4}{5}, \frac{5}{7}, \frac{6}{9}, \ldots$

54. $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \ldots$

55. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

56. $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$

57. $1, -1, 1, -1, 1, \ldots$

58. $1, 2, \frac{2^2}{2}, \frac{2^3}{6}, \frac{2^4}{24}, \frac{2^5}{120}, \ldots$

59. $1, 3, 1, 3, 1, \ldots$

60. $3, \frac{3}{2}, 1, \frac{3}{4}, \frac{3}{5}, \ldots$

61. $1 + \frac{1}{1}, 1 + \frac{1}{2}, 1 + \frac{1}{3}, 1 + \frac{1}{4}, 1 + \frac{1}{5}, \ldots$

62. $1 + \frac{1}{2}, 1 + \frac{3}{4}, 1 + \frac{7}{8}, 1 + \frac{15}{16}, 1 + \frac{31}{32}, \ldots$

In Exercises 63–66, write the first five terms of the sequence defined recursively.

63. $a_1 = 28, \quad a_{k+1} = a_k - 4$

64. $a_1 = 15, \quad a_{k+1} = a_k + 3$

65. $a_1 = 3, \quad a_{k+1} = 2(a_k - 1)$

66. $a_1 = 32, \quad a_{k+1} = \frac{1}{2}a_k$

In Exercises 67–70, write the first five terms of the sequence defined recursively. Use the pattern to write the nth term of the sequence as a function of n. (Assume that n begins with 1.)

67. $a_1 = 6, \quad a_{k+1} = a_k + 2$

68. $a_1 = 25, \quad a_{k+1} = a_k - 5$

69. $a_1 = 81, \quad a_{k+1} = \frac{1}{3}a_k$

70. $a_1 = 14, \quad a_{k+1} = (-2)a_k$

In Exercises 71–76, write the first five terms of the sequence. (Assume that n begins with 0.)

71. $a_n = \dfrac{1}{n!}$

72. $a_n = \dfrac{n!}{2n + 1}$

73. $a_n = \dfrac{1}{(n + 1)!}$

74. $a_n = \dfrac{n^2}{(n + 1)!}$

75. $a_n = \dfrac{(-1)^{2n}}{(2n)!}$

76. $a_n = \dfrac{(-1)^{2n+1}}{(2n + 1)!}$

In Exercises 77–84, simplify the factorial expression.

77. $\dfrac{4!}{6!}$

78. $\dfrac{5!}{8!}$

79. $\dfrac{12!}{4! \cdot 8!}$

80. $\dfrac{10! \cdot 3!}{4! \cdot 6!}$

81. $\dfrac{(n + 1)!}{n!}$

82. $\dfrac{(n + 2)!}{n!}$

83. $\dfrac{(2n - 1)!}{(2n + 1)!}$

84. $\dfrac{(3n + 1)!}{(3n)!}$

In Exercises 85–96, find the sum.

85. $\displaystyle\sum_{i=1}^{5} (2i + 1)$

86. $\displaystyle\sum_{i=1}^{6} (3i - 1)$

87. $\displaystyle\sum_{k=1}^{4} 10$

88. $\displaystyle\sum_{k=1}^{5} 6$

89. $\displaystyle\sum_{i=0}^{4} i^2$

90. $\displaystyle\sum_{i=0}^{5} 3i^2$

91. $\displaystyle\sum_{k=0}^{3} \frac{1}{k^2 + 1}$

92. $\displaystyle\sum_{j=3}^{5} \frac{1}{j^2 - 3}$

93. $\displaystyle\sum_{k=2}^{5} (k + 1)^2(k - 3)$

94. $\displaystyle\sum_{i=1}^{4} [(i - 1)^2 + (i + 1)^3]$

95. $\displaystyle\sum_{i=1}^{4} 2^i$

96. $\displaystyle\sum_{j=0}^{4} (-2)^j$

In Exercises 97–102, use a calculator to find the sum.

97. $\displaystyle\sum_{n=0}^{5} \frac{1}{2n + 1}$

98. $\displaystyle\sum_{j=1}^{10} \frac{3}{j + 1}$

99. $\displaystyle\sum_{k=0}^{4} \frac{(-1)^k}{k + 1}$

100. $\displaystyle\sum_{k=0}^{4} \frac{(-1)^k}{k!}$

101. $\displaystyle\sum_{n=0}^{25} \frac{1}{4^n}$

102. $\displaystyle\sum_{n=0}^{25} \frac{1}{5^{n+1}}$

In Exercises 103–112, use sigma notation to write the sum.

103. $\dfrac{1}{3(1)} + \dfrac{1}{3(2)} + \dfrac{1}{3(3)} + \cdots + \dfrac{1}{3(9)}$

104. $\dfrac{5}{1 + 1} + \dfrac{5}{1 + 2} + \dfrac{5}{1 + 3} + \cdots + \dfrac{5}{1 + 15}$

105. $\left[2\left(\frac{1}{8}\right) + 3\right] + \left[2\left(\frac{2}{8}\right) + 3\right] + \cdots + \left[2\left(\frac{8}{8}\right) + 3\right]$

106. $\left[1 - \left(\frac{1}{6}\right)^2\right] + \left[1 - \left(\frac{2}{6}\right)^2\right] + \cdots + \left[1 - \left(\frac{6}{6}\right)^2\right]$

107. $3 - 9 + 27 - 81 + 243 - 729$

108. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots - \frac{1}{128}$

109. $\dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \cdots - \dfrac{1}{20^2}$

110. $\dfrac{1}{1 \cdot 3} + \dfrac{1}{2 \cdot 4} + \dfrac{1}{3 \cdot 5} + \cdots + \dfrac{1}{10 \cdot 12}$

111. $\frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \frac{15}{32} + \frac{31}{64}$

112. $\frac{1}{2} + \frac{2}{4} + \frac{6}{8} + \frac{24}{16} + \frac{120}{32} + \frac{720}{64}$

In Exercises 113–116, find the indicated partial sum of the series.

113. $\displaystyle\sum_{i=1}^{\infty} 5\left(\frac{1}{2}\right)^i$

Fourth partial sum

114. $\displaystyle\sum_{i=1}^{\infty} 2\left(\frac{1}{3}\right)^i$

Fifth partial sum

115. $\displaystyle\sum_{n=1}^{\infty} 4\left(-\frac{1}{2}\right)^n$

Third partial sum

116. $\displaystyle\sum_{n=1}^{\infty} 8\left(-\frac{1}{4}\right)^n$

Fourth partial sum

In Exercises 117–120, find the sum of the infinite series.

117. $\sum_{i=1}^{\infty} 6\left(\frac{1}{10}\right)^i$

118. $\sum_{k=1}^{\infty} \left(\frac{1}{10}\right)^k$

119. $\sum_{k=1}^{\infty} 7\left(\frac{1}{10}\right)^k$

120. $\sum_{i=1}^{\infty} 2\left(\frac{1}{10}\right)^i$

121. COMPOUND INTEREST You deposit \$25,000 in an account that earns 7% interest compounded monthly. The balance in the account after n months is given by

$$A_n = 25{,}000\left(1 + \frac{0.07}{12}\right)^n, \quad n = 1, 2, 3, \ldots.$$

(a) Write the first six terms of the sequence.

(b) Find the balance in the account after 5 years by computing the 60th term of the sequence.

(c) Is the balance after 10 years twice the balance after 5 years? Explain.

122. COMPOUND INTEREST A deposit of \$10,000 is made in an account that earns 8.5% interest compounded quarterly. The balance in the account after n quarters is given by

$$A_n = 10{,}000\left(1 + \frac{0.085}{4}\right)^n, \quad n = 1, 2, 3, \ldots.$$

(a) Write the first eight terms of the sequence.

(b) Find the balance in the account after 10 years by computing the 40th term of the sequence.

(c) Is the balance after 20 years twice the balance after 10 years? Explain.

123. DATA ANALYSIS: NUMBER OF STORES The table shows the numbers a_n of Best Buy stores from 2002 through 2007. (Source: Best Buy Company, Inc.)

Year	Number of stores, a_n
2002	548
2003	595
2004	668
2005	786
2006	822
2007	923

(a) Use the *regression* feature of a graphing utility to find a linear sequence that models the data. Let n represent the year, with $n = 2$ corresponding to 2002.

(b) Use the *regression* feature of a graphing utility to find a quadratic sequence that models the data.

(c) Evaluate the sequences from parts (a) and (b) for $n = 2, 3, \ldots, 7$. Compare these values with those shown in the table. Which model is a better fit for the data? Explain.

(d) Which model do you think would better predict the number of Best Buy stores in the future? Use the model you chose to predict the number of Best Buy stores in 2013.

124. MEDICINE The numbers a_n (in thousands) of AIDS cases reported from 2000 through 2007 can be approximated by the model

$$a_n = 0.0768n^3 - 3.150n^2 + 41.56n - 136.4,$$
$$n = 10, 11, \ldots, 17$$

where n is the year, with $n = 10$ corresponding to 2000. (Source: U.S. Centers for Disease Control and Prevention)

(a) Find the terms of this finite sequence. Use the *statistical plotting* feature of a graphing utility to construct a bar graph that represents the sequence.

(b) What does the graph in part (a) say about reported cases of AIDS?

125. FEDERAL DEBT From 1995 to 2007, the federal debt of the United States rose from almost \$5 trillion to almost \$9 trillion. The federal debt a_n (in billions of dollars) from 1995 through 2007 is approximated by the model

$$a_n = 1.0904n^3 - 6.348n^2 + 41.76n + 4871.3,$$
$$n = 5, 6, \ldots, 17$$

where n is the year, with $n = 5$ corresponding to 1995. (Source: U.S. Office of Management and Budget)

(a) Find the terms of this finite sequence. Use the *statistical plotting* feature of a graphing utility to construct a bar graph that represents the sequence.

(b) What does the pattern in the bar graph in part (a) say about the future of the federal debt?

126. REVENUE The revenues a_n (in millions of dollars) of Amazon.com from 2001 through 2008 are shown in the figure on the next page. The revenues can be approximated by the model

$$a_n = 296.477n^2 - 469.11n + 3606.2,$$
$$n = 1, 2, \ldots, 8$$

where n is the year, with $n = 1$ corresponding to 2001. Use this model to approximate the total revenue from 2001 through 2008. Compare this sum with the result of adding the revenues shown in the figure on the next page. (Source: Amazon.com)