6.3 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. A ________ ________ ________ can be used to represent a quantity that involves both magnitude and direction.

2. The directed line segment $\overrightarrow{PQ}$ has ________ point P and ________ point Q.

3. The ________ of the directed line segment $\overrightarrow{PQ}$ is denoted by $\|\overrightarrow{PQ}\|$.

4. The set of all directed line segments that are equivalent to a given directed line segment $\overrightarrow{PQ}$ is a ________ $\mathbf{v}$ in the plane.

5. In order to show that two vectors are equivalent, you must show that they have the same ________ and the same ________ .

6. The directed line segment whose initial point is the origin is said to be in ________ ________ .

7. A vector that has a magnitude of 1 is called a ________ ________ .

8. The two basic vector operations are scalar ________ and vector ________ .

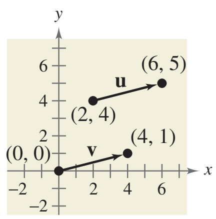
9. The vector $\mathbf{u} + \mathbf{v}$ is called the ________ of vector addition.

10. The vector sum $v_1\mathbf{i} + v_2\mathbf{j}$ is called a ________ ________ of the vectors $\mathbf{i}$ and $\mathbf{j}$, and the scalars v_1 and v_2 are called the ________ and ________ components of $\mathbf{v}$, respectively.

SKILLS AND APPLICATIONS

In Exercises 11 and 12, show that **u** and **v** are equivalent.

11.



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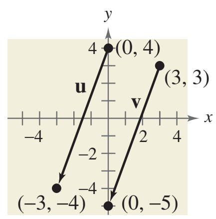

In Exercises 13–24, find the component form and the magnitude of the vector **v**.

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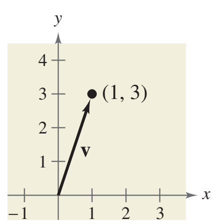

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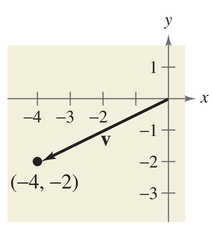

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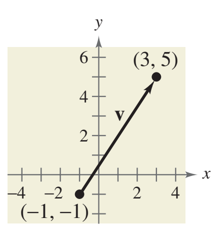

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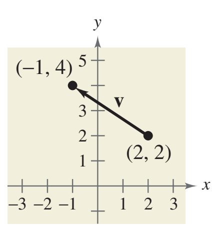

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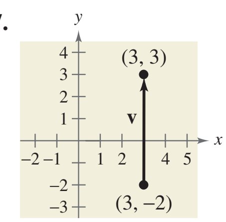

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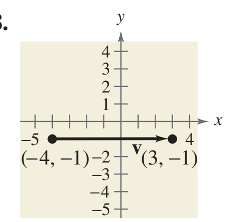
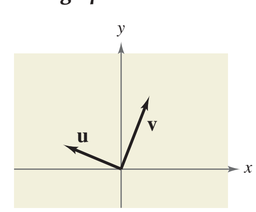


	Initial Point	*Terminal Point*
19.	$(-3, -5)$	$(5, 1)$
20.	$(-2, 7)$	$(5, -17)$
21.	$(1, 3)$	$(-8, -9)$
22.	$(1, 11)$	$(9, 3)$
23.	$(-1, 5)$	$(15, 12)$
24.	$(-3, 11)$	$(9, 40)$

In Exercises 25–30, use the figure to sketch a graph of the specified vector. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.



25. $-\mathbf{v}$

26. $5\mathbf{v}$

27. $\mathbf{u} + \mathbf{v}$

28. $\mathbf{u} + 2\mathbf{v}$

29. $\mathbf{u} - \mathbf{v}$

30. $\mathbf{v} - \frac{1}{2}\mathbf{u}$

In Exercises 31–38, find (a) $\mathbf{u} + \mathbf{v}$, (b) $\mathbf{u} - \mathbf{v}$, and (c) $2\mathbf{u} - 3\mathbf{v}$, Then sketch each resultant vector.

31. $\mathbf{u} = \langle 2, 1 \rangle$, $\mathbf{v} = \langle 1, 3 \rangle$
32. $\mathbf{u} = \langle 2, 3 \rangle$, $\mathbf{v} = \langle 4, 0 \rangle$
33. $\mathbf{u} = \langle -5, 3 \rangle$, $\mathbf{v} = \langle 0, 0 \rangle$
34. $\mathbf{u} = \langle 0, 0 \rangle$, $\mathbf{v} = \langle 2, 1 \rangle$
35. $\mathbf{u} = \mathbf{i} + \mathbf{j}$, $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$
36. $\mathbf{u} = -2\mathbf{i} + \mathbf{j}$, $\mathbf{v} = 3\mathbf{j}$
37. $\mathbf{u} = 2\mathbf{i}$, $\mathbf{v} = \mathbf{j}$
38. $\mathbf{u} = 2\mathbf{j}$, $\mathbf{v} = 3\mathbf{i}$

In Exercises 39–48, find a unit vector in the direction of the given vector. Verify that the result has a magnitude of 1.

39. $\mathbf{u} = \langle 3, 0 \rangle$
40. $\mathbf{u} = \langle 0, -2 \rangle$
41. $\mathbf{v} = \langle -2, 2 \rangle$
42. $\mathbf{v} = \langle 5, -12 \rangle$
43. $\mathbf{v} = \mathbf{i} + \mathbf{j}$
44. $\mathbf{v} = 6\mathbf{i} - 2\mathbf{j}$
45. $\mathbf{w} = 4\mathbf{j}$
46. $\mathbf{w} = -6\mathbf{i}$
47. $\mathbf{w} = \mathbf{i} - 2\mathbf{j}$
48. $\mathbf{w} = 7\mathbf{j} - 3\mathbf{i}$

In Exercises 49–52, find the vector $\mathbf{v}$ with the given magnitude and the same direction as $\mathbf{u}$.

	Magnitude	*Direction*
49.	$\|\mathbf{v}\| = 10$	$\mathbf{u} = \langle -3, 4 \rangle$
50.	$\|\mathbf{v}\| = 3$	$\mathbf{u} = \langle -12, -5 \rangle$
51.	$\|\mathbf{v}\| = 9$	$\mathbf{u} = \langle 2, 5 \rangle$
52.	$\|\mathbf{v}\| = 8$	$\mathbf{u} = \langle 3, 3 \rangle$

In Exercises 53–56, the initial and terminal points of a vector are given. Write a linear combination of the standard unit vectors $\mathbf{i}$ and $\mathbf{j}$.

	Initial Point	*Terminal Point*
53.	$(-2, 1)$	$(3, -2)$
54.	$(0, -2)$	$(3, 6)$
55.	$(-6, 4)$	$(0, 1)$
56.	$(-1, -5)$	$(2, 3)$

In Exercises 57–62, find the component form of $\mathbf{v}$ and sketch the specified vector operations geometrically, where $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$, and $\mathbf{w} = \mathbf{i} + 2\mathbf{j}$.

57. $\mathbf{v} = \frac{3}{2}\mathbf{u}$
58. $\mathbf{v} = \frac{3}{4}\mathbf{w}$
59. $\mathbf{v} = \mathbf{u} + 2\mathbf{w}$
60. $\mathbf{v} = -\mathbf{u} + \mathbf{w}$
61. $\mathbf{v} = \frac{1}{2}(3\mathbf{u} + \mathbf{w})$
62. $\mathbf{v} = \mathbf{u} - 2\mathbf{w}$

In Exercises 63–66, find the magnitude and direction angle of the vector $\mathbf{v}$.

63. $\mathbf{v} = 6\mathbf{i} - 6\mathbf{j}$
64. $\mathbf{v} = -5\mathbf{i} + 4\mathbf{j}$
65. $\mathbf{v} = 3(\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$
66. $\mathbf{v} = 8(\cos 135°\mathbf{i} + \sin 135°\mathbf{j})$

In Exercises 67–74, find the component form of $\mathbf{v}$ given its magnitude and the angle it makes with the positive x-axis. Sketch $\mathbf{v}$.

	Magnitude	*Angle*
67.	$\|\mathbf{v}\| = 3$	$\theta = 0°$
68.	$\|\mathbf{v}\| = 1$	$\theta = 45°$
69.	$\|\mathbf{v}\| = \frac{7}{2}$	$\theta = 150°$
70.	$\|\mathbf{v}\| = \frac{3}{4}$	$\theta = 150°$
71.	$\|\mathbf{v}\| = 2\sqrt{3}$	$\theta = 45°$
72.	$\|\mathbf{v}\| = 4\sqrt{3}$	$\theta = 90°$
73.	$\|\mathbf{v}\| = 3$	$\mathbf{v}$ in the direction $3\mathbf{i} + 4\mathbf{j}$
74.	$\|\mathbf{v}\| = 2$	$\mathbf{v}$ in the direction $\mathbf{i} + 3\mathbf{j}$

In Exercises 75–78, find the component form of the sum of $\mathbf{u}$ and $\mathbf{v}$ with direction angles $\theta_{\mathbf{u}}$ and $\theta_{\mathbf{v}}$.

	Magnitude	*Angle*
75.	$\|\mathbf{u}\| = 5$	$\theta_{\mathbf{u}} = 0°$
	$\|\mathbf{v}\| = 5$	$\theta_{\mathbf{v}} = 90°$
76.	$\|\mathbf{u}\| = 4$	$\theta_{\mathbf{u}} = 60°$
	$\|\mathbf{v}\| = 4$	$\theta_{\mathbf{v}} = 90°$
77.	$\|\mathbf{u}\| = 20$	$\theta_{\mathbf{u}} = 45°$
	$\|\mathbf{v}\| = 50$	$\theta_{\mathbf{v}} = 180°$
78.	$\|\mathbf{u}\| = 50$	$\theta_{\mathbf{u}} = 30°$
	$\|\mathbf{v}\| = 30$	$\theta_{\mathbf{v}} = 110°$

In Exercises 79 and 80, use the Law of Cosines to find the angle α between the vectors. (Assume $0° \le \alpha \le 180°$.)

79. $\mathbf{v} = \mathbf{i} + \mathbf{j}$, $\mathbf{w} = 2\mathbf{i} - 2\mathbf{j}$
80. $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$, $\mathbf{w} = 2\mathbf{i} - \mathbf{j}$

RESULTANT FORCE In Exercises 81 and 82, find the angle between the forces given the magnitude of their resultant. (*Hint:* Write force 1 as a vector in the direction of the positive x-axis and force 2 as a vector at an angle θ with the positive x-axis.)

	Force 1	*Force 2*	*Resultant Force*
81.	45 pounds	60 pounds	90 pounds
82.	3000 pounds	1000 pounds	3750 pounds

83. VELOCITY A gun with a muzzle velocity of 1200 feet per second is fired at an angle of 6° above the horizontal. Find the vertical and horizontal components of the velocity.

84. Detroit Tigers pitcher Joel Zumaya was recorded throwing a pitch at a velocity of 104 miles per hour. If he threw the pitch at an angle of 35° below the horizontal, find the vertical and horizontal components of the velocity. (Source: Damon Lichtenwalner, Baseball Info Solutions)

85. RESULTANT FORCE Forces with magnitudes of 125 newtons and 300 newtons act on a hook (see figure). The angle between the two forces is 45°. Find the direction and magnitude of the resultant of these forces.

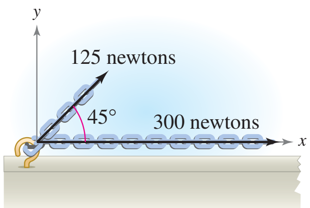


FIGURE FOR 85

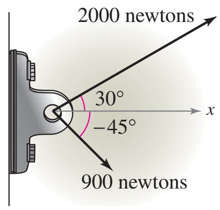


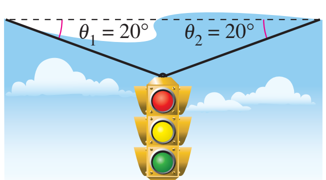
FIGURE FOR 86

86. RESULTANT FORCE Forces with magnitudes of 2000 newtons and 900 newtons act on a machine part at angles of 30° and −45°, respectively, with the x-axis (see figure). Find the direction and magnitude of the resultant of these forces.

87. RESULTANT FORCE Three forces with magnitudes of 75 pounds, 100 pounds, and 125 pounds act on an object at angles of 30°, 45°, and 120°, respectively, with the positive x-axis. Find the direction and magnitude of the resultant of these forces.

88. RESULTANT FORCE Three forces with magnitudes of 70 pounds, 40 pounds, and 60 pounds act on an object at angles of −30°, 45°, and 135°, respectively, with the positive x-axis. Find the direction and magnitude of the resultant of these forces.

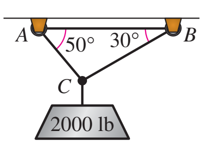
89. A traffic light weighing 12 pounds is suspended by two cables (see figure). Find the tension in each cable.



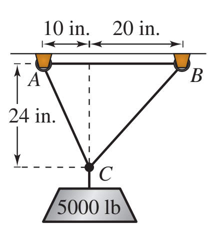
90. Repeat Exercise 89 if $\theta_1 = 40°$ and $\theta_2 = 35°$.

CABLE TENSION **In Exercises 91 and 92, use the figure to determine the tension in each cable supporting the load.**

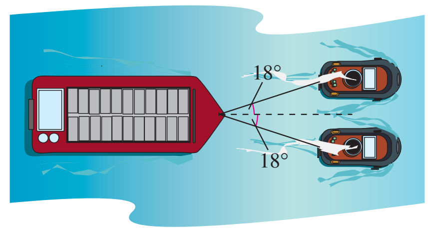
91.



92.



93. TOW LINE TENSION A loaded barge is being towed by two tugboats, and the magnitude of the resultant is 6000 pounds directed along the axis of the barge (see figure). Find the tension in the tow lines if they each make an 18° angle with the axis of the barge.



94. ROPE TENSION To carry a 100-pound cylindrical weight, two people lift on the ends of short ropes that are tied to an eyelet on the top center of the cylinder. Each rope makes a 20° angle with the vertical. Draw a figure that gives a visual representation of the situation, and find the tension in the ropes.

In Exercises 95–98, a force of F pounds is required to pull an object weighing W pounds up a ramp inclined at θ degrees from the horizontal.

95. Find F if $W = 100$ pounds and $\theta = 12°$.

96. Find W if $F = 600$ pounds and $\theta = 14°$.

97. Find θ if $F = 5000$ pounds and $W = 15{,}000$ pounds.

98. Find F if $W = 5000$ pounds and $\theta = 26°$.

99. WORK A heavy object is pulled 30 feet across a floor, using a force of 100 pounds. The force is exerted at an angle of 50° above the horizontal (see figure). Find the work done. (Use the formula for work, $W = FD$, where F is the component of the force in the direction of motion and D is the distance.)

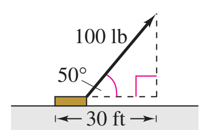


FIGURE FOR 99

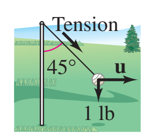


FIGURE FOR 100

100. ROPE TENSION A tetherball weighing 1 pound is pulled outward from the pole by a horizontal force **u** until the rope makes a 45° angle with the pole (see figure). Determine the resulting tension in the rope and the magnitude of **u**.