6.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. An ________ triangle is a triangle that has no right angle.

2. For triangle ABC, the Law of Sines is given by $\dfrac{a}{\sin A} = $ ________ $= \dfrac{c}{\sin C}$.

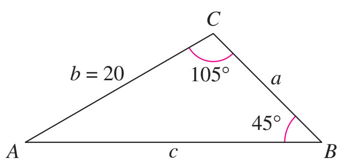
3. Two ________ and one ________ determine a unique triangle.

4. The area of an oblique triangle is given by $\frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = $ ________ .

SKILLS AND APPLICATIONS

In Exercises 5–24, use the Law of Sines to solve the triangle. Round your answers to two decimal places.

5.



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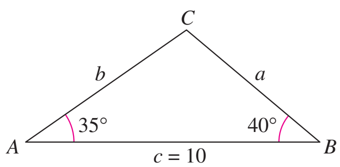

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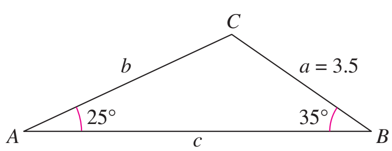

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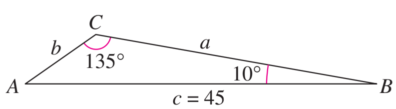


9. $A = 102.4°,\quad C = 16.7°,\quad a = 21.6$

10. $A = 24.3°,\quad C = 54.6°,\quad c = 2.68$

11. $A = 83° 20',\quad C = 54.6°,\quad c = 18.1$

12. $A = 5° 40',\quad B = 8° 15',\quad b = 4.8$

13. $A = 35°,\quad B = 65°,\quad c = 10$

14. $A = 120°,\quad B = 45°,\quad c = 16$

15. $A = 55°,\quad B = 42°,\quad c = \frac{3}{4}$

16. $B = 28°,\quad C = 104°,\quad a = 3\frac{5}{8}$

17. $A = 36°,\quad a = 8,\quad b = 5$

18. $A = 60°,\quad a = 9,\quad c = 10$

19. $B = 15° 30',\quad a = 4.5,\quad b = 6.8$

20. $B = 2° 45',\quad b = 6.2,\quad c = 5.8$

21. $A = 145°,\quad a = 14,\quad b = 4$

22. $A = 100°,\quad a = 125,\quad c = 10$

23. $A = 110° 15',\quad a = 48,\quad b = 16$

24. $C = 95.20°,\quad a = 35,\quad c = 50$

In Exercises 25–34, use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places.

25. $A = 110°,\quad a = 125,\quad b = 100$

26. $A = 110°,\quad a = 125,\quad b = 200$

27. $A = 76°,\quad a = 18,\quad b = 20$

28. $A = 76°,\quad a = 34,\quad b = 21$

29. $A = 58°,\quad a = 11.4,\quad b = 12.8$

30. $A = 58°,\quad a = 4.5,\quad b = 12.8$

31. $A = 120°,\quad a = b = 25$

32. $A = 120°,\quad a = 25,\quad b = 24$

33. $A = 45°,\quad a = b = 1$

34. $A = 25° 4',\quad a = 9.5,\quad b = 22$

In Exercises 35–38, find values for b such that the triangle has (a) one solution, (b) two solutions, and (c) no solution.

35. $A = 36°,\quad a = 5$

36. $A = 60°,\quad a = 10$

37. $A = 10°,\quad a = 10.8$

38. $A = 88°,\quad a = 315.6$

In Exercises 39–44, find the area of the triangle having the indicated angle and sides.

39. $C = 120°,\quad a = 4,\quad b = 6$

40. $B = 130°,\quad a = 62,\quad c = 20$

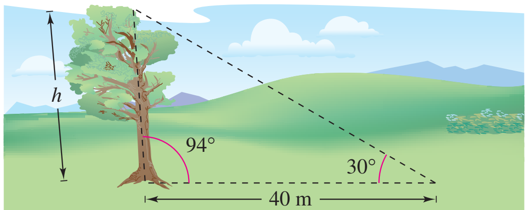
41. $A = 43° 45',\quad b = 57,\quad c = 85$

42. $A = 5° 15',\quad b = 4.5,\quad c = 22$

43. $B = 72° 30',\quad a = 105,\quad c = 64$

44. $C = 84° 30',\quad a = 16,\quad b = 20$

45. HEIGHT Because of prevailing winds, a tree grew so that it was leaning 4° from the vertical. At a point 40 meters from the tree, the angle of elevation to the top of the tree is 30° (see figure). Find the height h of the tree.



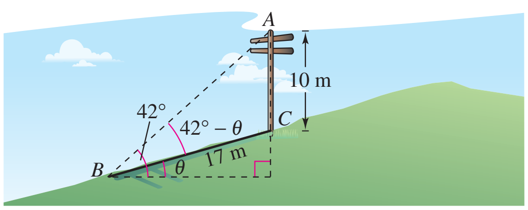
46. HEIGHT A flagpole at a right angle to the horizontal is located on a slope that makes an angle of 12° with the horizontal. The flagpole's shadow is 16 meters long and points directly up the slope. The angle of elevation from the tip of the shadow to the sun is 20°.

(a) Draw a triangle to represent the situation. Show the known quantities on the triangle and use a variable to indicate the height of the flagpole.

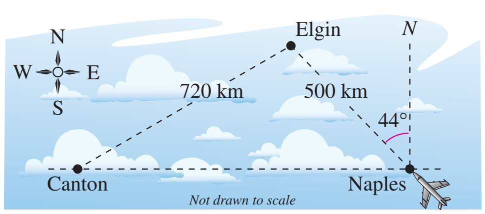
(b) Write an equation that can be used to find the height of the flagpole.

(c) Find the height of the flagpole.

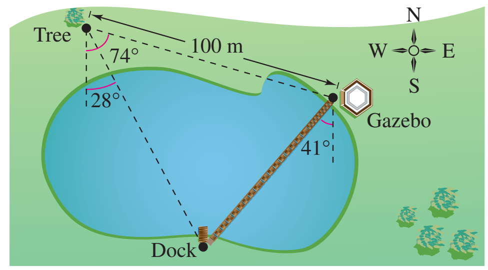
47. ANGLE OF ELEVATION A 10-meter utility pole casts a 17-meter shadow directly down a slope when the angle of elevation of the sun is 42° (see figure). Find θ, the angle of elevation of the ground.



48. FLIGHT PATH A plane flies 500 kilometers with a bearing of 316° from Naples to Elgin (see figure). The plane then flies 720 kilometers from Elgin to Canton (Canton is due west of Naples). Find the bearing of the flight from Elgin to Canton.



49. BRIDGE DESIGN A bridge is to be built across a small lake from a gazebo to a dock (see figure). The bearing from the gazebo to the dock is S 41° W. From a tree 100 meters from the gazebo, the bearings to the gazebo and the dock are S 74° E and S 28° E, respectively. Find the distance from the gazebo to the dock.



50. RAILROAD TRACK DESIGN The circular arc of a railroad curve has a chord of length 3000 feet corresponding to a central angle of 40°.

(a) Draw a diagram that visually represents the situation. Show the known quantities on the diagram and use the variables r and s to represent the radius of the arc and the length of the arc, respectively.

(b) Find the radius r of the circular arc.

(c) Find the length s of the circular arc.

51. GLIDE PATH A pilot has just started on the glide path for landing at an airport with a runway of length 9000 feet. The angles of depression from the plane to the ends of the runway are 17.5° and 18.8°.

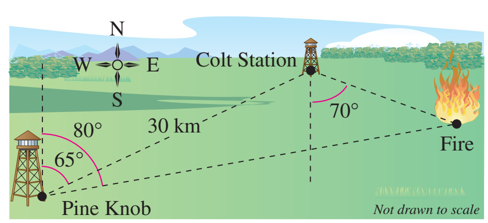
(a) Draw a diagram that visually represents the situation.

(b) Find the air distance the plane must travel until touching down on the near end of the runway.

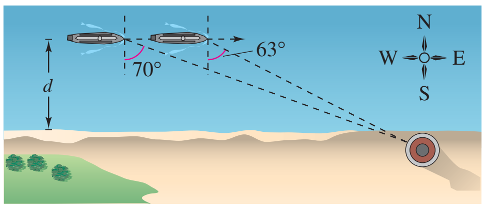
(c) Find the ground distance the plane must travel until touching down.

(d) Find the altitude of the plane when the pilot begins the descent.

52. LOCATING A FIRE The bearing from the Pine Knob fire tower to the Colt Station fire tower is N 65° E, and the two towers are 30 kilometers apart. A fire spotted by rangers in each tower has a bearing of N 80° E from Pine Knob and S 70° E from Colt Station (see figure). Find the distance of the fire from each tower.



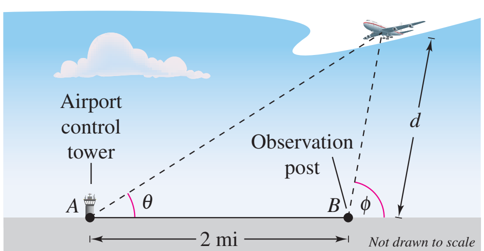
53. DISTANCE A boat is sailing due east parallel to the shoreline at a speed of 10 miles per hour. At a given time, the bearing to the lighthouse is S 70° E, and 15 minutes later the bearing is S 63° E (see figure). The lighthouse is located at the shoreline. What is the distance from the boat to the shoreline?



54. DISTANCE A family is traveling due west on a road that passes a famous landmark. At a given time the bearing to the landmark is N 62° W, and after the family travels 5 miles farther the bearing is N 38° W. What is the closest the family will come to the landmark while on the road?

55. ALTITUDE The angles of elevation to an airplane from two points A and B on level ground are 55° and 72°, respectively. The points A and B are 2.2 miles apart, and the airplane is east of both points in the same vertical plane. Find the altitude of the plane.

56. DISTANCE The angles of elevation θ and ϕ to an airplane from the airport control tower and from an observation post 2 miles away are being continuously monitored (see figure). Write an equation giving the distance d between the plane and observation post in terms of θ and ϕ.



EXPLORATION

TRUE OR FALSE? **In Exercises 57–59, determine whether the statement is true or false. Justify your answer.**

57. If a triangle contains an obtuse angle, then it must be oblique.

58. Two angles and one side of a triangle do not necessarily determine a unique triangle.

59. If three sides or three angles of an oblique triangle are known, then the triangle can be solved.

60. GRAPHICAL AND NUMERICAL ANALYSIS In the figure, α and β are positive angles.

(a) Write α as a function of β.

(b) Use a graphing utility to graph the function in part (a). Determine its domain and range.

(c) Use the result of part (a) to write c as a function of β.

(d) Use a graphing utility to graph the function in part (c). Determine its domain and range.

(e) Complete the table. What can you infer?

β	0.4	0.8	1.2	1.6	2.0	2.4	2.8
α							
c							

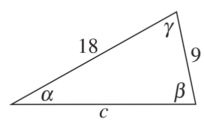


FIGURE FOR 60

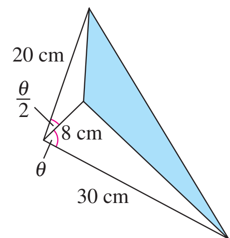


FIGURE FOR 61

61. GRAPHICAL ANALYSIS

(a) Write the area A of the shaded region in the figure as a function of θ.

(b) Use a graphing utility to graph the function.

(c) Determine the domain of the function. Explain how the area of the region and the domain of the function would change if the eight-centimeter line segment were decreased in length.

62. CAPSTONE In the figure, a triangle is to be formed by drawing a line segment of length a from (4, 3) to the positive x-axis. For what value(s) of a can you form (a) one triangle, (b) two triangles, and (c) no triangles? Explain your reasoning.