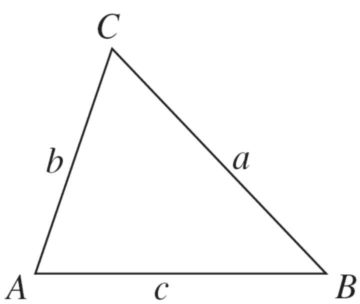
6.1 LAW OF SINES

Introduction

Introduction

Oblique triangles—triangles that have no right angles.



Law of Sine

1. Two angles and any side (AAS or ASA)

2. Two sides and an angle opposite one of them (SSA)

Law of Cosine

3. Three sides (SSS)

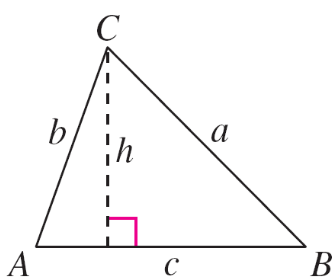
4. Two sides and their included angle (SAS)

Introduction

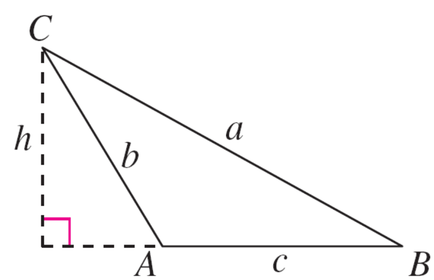
Law of Sines

If ABC is a triangle with sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$



A is acute.

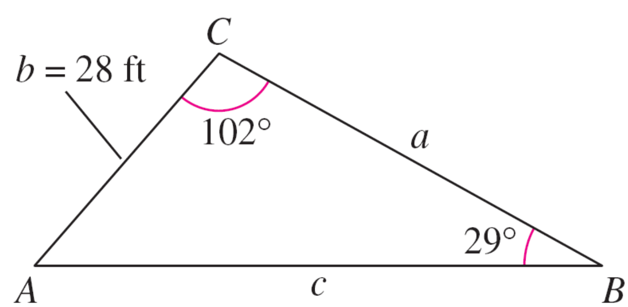


A is obtuse.

Example 1 – *Given Two Angles and One Side—AAS*

For the triangle in figure, $C = 120°$, $B = 29°$, and $b = 28$ feet. Find the remaining angle and sides.

Solution:



The third angle of the triangle is

$$A = 180° - B - C$$

$$= 180° - 29° - 102°$$

$$= 49°.$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a = \frac{b}{\sin B}(\sin A) = \frac{28}{\sin 29°}(\sin 49°) \approx 43.59 \text{ feet}$$

$$c = \frac{b}{\sin B}(\sin C) = \frac{28}{\sin 29°}(\sin 102°) \approx 56.49 \text{ feet.}$$

Example 1 – *Solution*

The third angle of the triangle is

$A = 180° – B – C$

$= 180° – 29° – 102°$

$= 49°.$

By the Law of Sines, you have $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a = \frac{b}{\sin B}(\sin A) = \frac{28}{\sin 29°}(\sin 49°) \approx 43.59 \text{ feet}$$

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The Ambiguous Case (SSA)

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In Examples 1, we saw that two angles and one side determine a unique triangle.

However, if two sides and one opposite angle are given, three possible situations can occur:

(1) no such triangle exists,

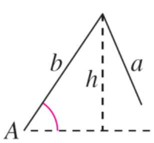
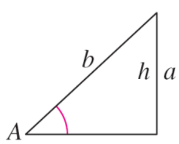
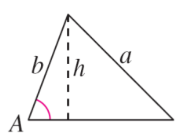
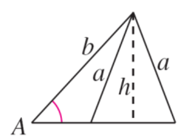
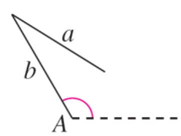
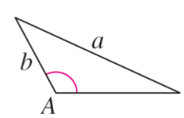
(2) one such triangle exists, or

(3) two distinct triangles may satisfy the conditions.

The Ambiguous Case (SSA)

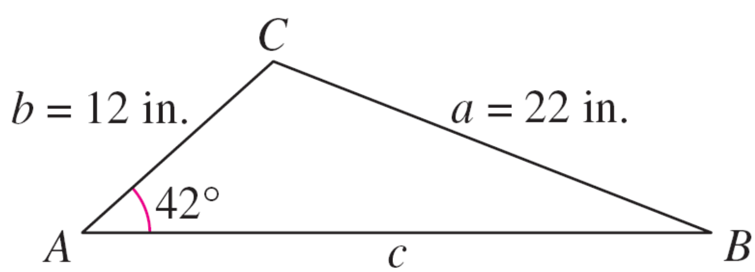
The Ambiguous Case (SSA)

Consider a triangle in which you are given a, b, and A. $(h = b \sin A)$

	A is acute.	A is acute.	A is acute.	A is acute.	A is obtuse.	A is obtuse.
Sketch						
Necessary condition	$a < h$	$a = h$	$a \geq b$	$h < a < b$	$a \leq b$	$a > b$
Triangles possible	None	One	One	Two	None	One

Example 3 – *Single-Solution Case—SSA*

For the triangle in Figure 6.4, a = 22 inches, b = 12 inches, and A = 42°. Find the remaining side and angles.



One solution: $a \geq b$

Figure 6.4

Example 3 – *Solution*

By the Law of Sines, you have

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$ Reciprocal form

$$\sin B = b\left(\frac{\sin A}{a}\right)$$ Multiply each side by *b*.

$$\sin B = 12\left(\frac{\sin 42^\circ}{22}\right)$$ Substitute for *A*, *a*, and *b*.

$$B \approx 21.41^\circ.$$ *B* is acute.

Example 3 – *Solution*

cont'd

Now, you can determine that

$$C \approx 180° - 42° - 21.41°$$

$$= 116.59°.$$

Then, the remaining side is

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

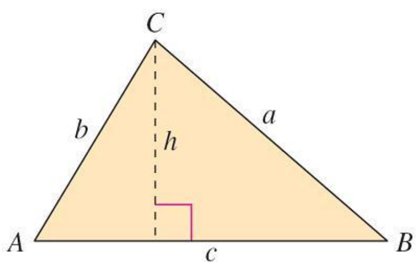
$$c = \frac{a}{\sin A}(\sin C) = \frac{22}{\sin 42°}(\sin 116.59°) \approx 29.40 \text{ inches.}$$

Area of an Oblique Triangle

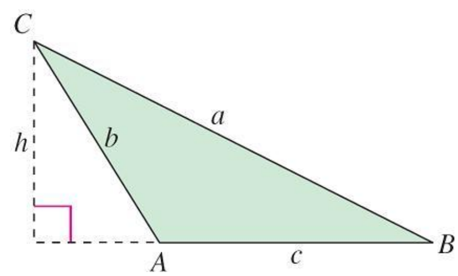
Area of an Oblique Triangle

The procedure used to prove the Law of Sines leads to a simple formula for the area of an oblique triangle.

Referring to Figure 6.7, note that each triangle has a height of $h = b \sin A$. Consequently, the area of each triangle is



A is acute.



A is obtuse.

Figure 6.7

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(c)(b \sin A) = \frac{1}{2} bc \sin A.$$

Area of an Oblique Triangle

By similar arguments, you can develop the formulas

$$\text{Area} = \frac{1}{2}\, ab \sin C = \frac{1}{2}\, ac \sin B.$$

Area of an Oblique Triangle

The area of any triangle is one-half the product of the lengths of two sides times the sine of their included angle. That is,

$$\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B.$$

Area of an Oblique Triangle

Note that if angle *A* is 90°, the formula gives the area for a right triangle:

$$\text{Area} = \frac{1}{2}bc(\sin 90^\circ) = \frac{1}{2}bc = \frac{1}{2}(\text{base})(\text{height}).$$

sin 90° = 1

Similar results are obtained for angles *C* and *B* equal to 90°.

Example 6 – *Finding the Area of a Triangular Lot*

Find the area of a triangular lot having two sides of lengths 90 meters and 52 meters and an included angle of 102°.

Solution:

Consider a = 90 meters, b = 52 meters, and angle C = 102°, as shown in Figure 6.8.

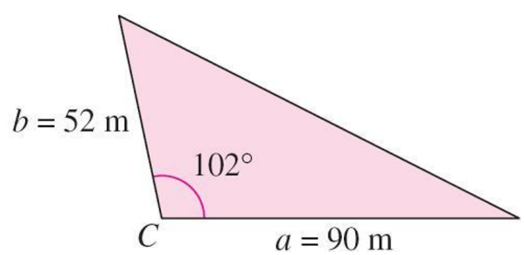


Figure 6.8

Then, the area of the triangle is

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$= \frac{1}{2}(90)(52)(\sin 102°)$$

$$\approx 2289 \text{ square meters.}$$

Application

Example 7 – *An Application of the Law of Sines*

The course for a boat race starts at point *A* in Figure 6.9 and proceeds in the direction S 52° W to point *B*, then in the direction S 40° E to point *C*, and finally back to *A.* Point *C* lies 8 kilometers directly south of point *A*. Approximate the total distance of the race course.

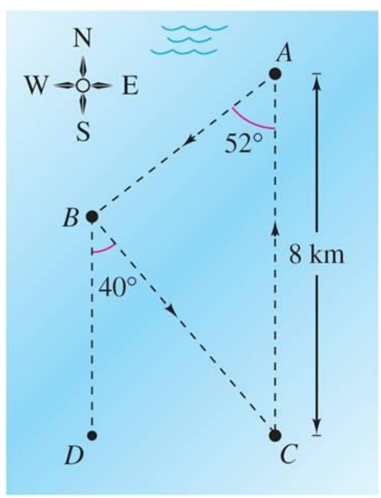


Figure 6.9

Example 7 – *Solution*

Because lines *BD* and *AC* are parallel, it follows that

$\angle BCA \cong \angle CBD$.

Consequently, triangle *ABC* has the measures shown in Figure 6.10.

The measure of angle *B* is
$180° – 52° – 40° = 88°$.

Using the Law of Sines,

$$\frac{a}{\sin 52°} = \frac{b}{\sin 88°} = \frac{c}{\sin 40°}.$$

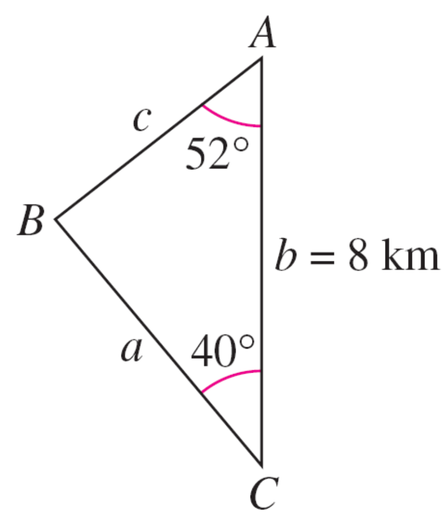


Figure 6.10

Example 7 – *Solution*

cont'd

Because $b = 8$,

$$a = \frac{8}{\sin 88^\circ}(\sin 52^\circ) \approx 6.308$$

and

$$c = \frac{8}{\sin 88^\circ}(\sin 40^\circ) \approx 5.145.$$

The total length of the course is approximately

Length ≈ 8 + 6.308 + 5.145

=19.453 kilometers.