EXERCISES FOR SECTION 4.5

In Exercises 1–6, complete the table by identifying u and du for the integral.

$\int f(g(x))g'(x)\,dx$	$u = g(x)$	$du = g'(x)\,dx$
1. $\int (5x^2 + 1)^2(10x)\,dx$		
2. $\int x^2\sqrt{x^3 + 1}\,dx$		
3. $\int \frac{x}{\sqrt{x^2 + 1}}\,dx$		
4. $\int \sec 2x \tan 2x\,dx$		
5. $\int \tan^2 x \sec^2 x\,dx$		
6. $\int \frac{\cos x}{\sin^2 x}\,dx$		

In Exercises 7–34, find the indefinite integral and check the result by differentiation.

7. $\int (1 + 2x)^4(2)\,dx$

8. $\int (x^2 - 9)^3(2x)\,dx$

9. $\int \sqrt{9 - x^2}\,(-2x)\,dx$

10. $\int \sqrt[3]{(1 - 2x^2)}(-4x)\,dx$

11. $\int x^3(x^4 + 3)^2\,dx$

12. $\int x^2(x^3 + 5)^4\,dx$

13. $\int x^2(x^3 - 1)^4\,dx$

14. $\int x(4x^2 + 3)^3\,dx$

15. $\int t\sqrt{t^2 + 2}\,dt$

16. $\int t^3\sqrt{t^4 + 5}\,dt$

17. $\int 5x\,\sqrt[3]{1 - x^2}\,dx$

18. $\int u^2\sqrt{u^3 + 2}\,du$

19. $\int \frac{x}{(1 - x^2)^3}\,dx$

20. $\int \frac{x^3}{(1 + x^4)^2}\,dx$

21. $\int \frac{x^2}{(1 + x^3)^2}\,dx$

22. $\int \frac{x^2}{(16 - x^3)^2}\,dx$

23. $\int \frac{x}{\sqrt{1 - x^2}}\,dx$

24. $\int \frac{x^3}{\sqrt{1 + x^4}}\,dx$

25. $\int \left(1 + \frac{1}{t}\right)^3\left(\frac{1}{t^2}\right)dt$

26. $\int \left[x^2 + \frac{1}{(3x)^2}\right]dx$

27. $\int \frac{1}{\sqrt{2x}}\,dx$

28. $\int \frac{1}{2\sqrt{x}}\,dx$

29. $\int \frac{x^2 + 3x + 7}{\sqrt{x}}\,dx$

30. $\int \frac{t + 2t^2}{\sqrt{t}}\,dt$

31. $\int t^2\left(t - \frac{2}{t}\right)dt$

32. $\int \left(\frac{t^3}{3} + \frac{1}{4t^2}\right)dt$

33. $\int (9 - y)\sqrt{y}\,dy$

34. $\int 2\pi y(8 - y^{3/2})\,dy$

In Exercises 35–38, solve the differential equation.

35. $\frac{dy}{dx} = 4x + \frac{4x}{\sqrt{16 - x^2}}$

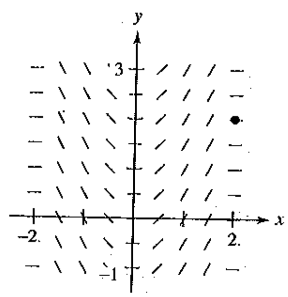
36. $\frac{dy}{dx} = \frac{10x^2}{\sqrt{1 + x^3}}$

37. $\frac{dy}{dx} = \frac{x + 1}{(x^2 + 2x - 3)^2}$

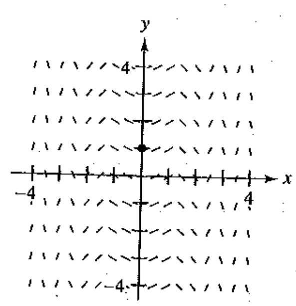
38. $\frac{dy}{dx} = \frac{x - 4}{\sqrt{x^2 - 8x + 1}}$

Slope Fields In Exercises 39 and 40, a differential equation, a point, and a slope field are given. A *slope field* consists of line segments with slopes given by the differential equation. These line segments give a visual perspective of the directions of the solutions of the differential equation. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the indicated point. (To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.) (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a).

39. $\frac{dy}{dx} = x\sqrt{4 - x^2}$, $(2, 2)$



40. $\frac{dy}{dx} = x\cos x^2$, $(0, 1)$



In Exercises 41–54, find the indefinite integral.

41. $\int \pi \sin \pi x\,dx$

42. $\int 4x^3 \sin x^4\,dx$

43. $\int \sin 2x\,dx$

44. $\int \cos 6x\,dx$

45. $\int \frac{1}{\theta^2}\cos\frac{1}{\theta}\,d\theta$

46. $\int x \sin x^2\,dx$

47. $\int \sin 2x \cos 2x\,dx$

48. $\int \sec(1 - x)\tan(1 - x)\,dx$

49. $\int \tan^4 x \sec^2 x\,dx$

50. $\int \sqrt{\tan x}\,\sec^2 x\,dx$

51. $\int \frac{\csc^2 x}{\cot^3 x}\,dx$

52. $\int \frac{\sin x}{\cos^3 x}\,dx$

53. $\int \cot^2 x\,dx$

54. $\int \csc^2\left(\frac{x}{2}\right)dx$

In Exercises 55 and 56, find an equation for the function f that has the indicated derivative and whose graph passes through the given point.

Derivative	*Point*
55. $f'(x) = \cos\frac{x}{2}$	$(0, 3)$
56. $f'(x) = \pi \sec \pi x \tan \pi x$	$\left(\frac{1}{3}, 1\right)$

In Exercises 57–64, find the indefinite integral by the method shown in Example 5.

57. $\int x\sqrt{x+2}\,dx$

58. $\int x\sqrt{2x+1}\,dx$

59. $\int x^2\sqrt{1-x}\,dx$

60. $\int (x+1)\sqrt{2-x}\,dx$

61. $\int \frac{x^2-1}{\sqrt{2x-1}}\,dx$

62. $\int \frac{2x+1}{\sqrt{x+4}}\,dx$

63. $\int \frac{-x}{(x+1)-\sqrt{x+1}}\,dx$

64. $\int t\sqrt[3]{t-4}\,dt$

In Exercises 65–76, evaluate the definite integral. Use a graphing utility to verify your result.

65. $\int_{-1}^{1} x(x^2+1)^3\,dx$

66. $\int_{-2}^{4} x^2(x^3+8)^2\,dx$

67. $\int_{1}^{2} 2x^2\sqrt{x^3+1}\,dx$

68. $\int_{0}^{1} x\sqrt{1-x^2}\,dx$

69. $\int_{0}^{4} \frac{1}{\sqrt{2x+1}}\,dx$

70. $\int_{0}^{2} \frac{x}{\sqrt{1+2x^2}}\,dx$

71. $\int_{1}^{9} \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}\,dx$

72. $\int_{0}^{2} x\sqrt[3]{4+x^2}\,dx$

73. $\int_{1}^{2} (x-1)\sqrt{2-x}\,dx$

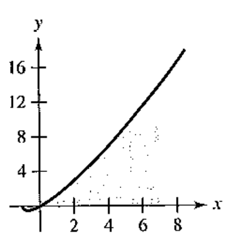
74. $\int_{1}^{5} \frac{x}{\sqrt{2x-1}}\,dx$

75. $\int_{0}^{\pi/2} \cos\left(\frac{2x}{3}\right)dx$

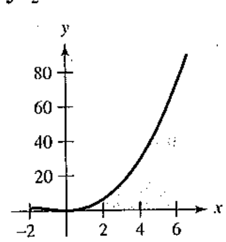
76. $\int_{\pi/3}^{\pi/2} (x+\cos x)\,dx$

In Exercises 77–82, find the area of the region. Use a graphing utility to verify your result.

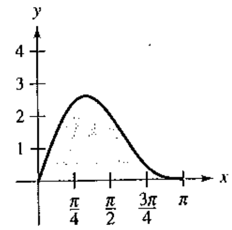
77. $\int_{0}^{7} x\sqrt[3]{x+1}\,dx$



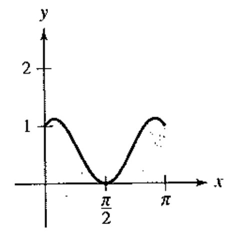
78. $\int_{-2}^{6} x^2\sqrt[3]{x+2}\,dx$



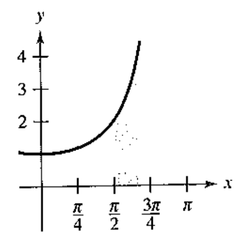
79. $y = 2\sin x + \sin 2x$



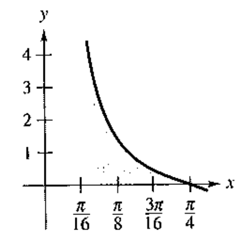
80. $y = \sin x + \cos 2x$



81. $\int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right)dx$



82. $\int_{\pi/12}^{\pi/4} \csc 2x \cot 2x\,dx$



In Exercises 83–88, use a graphing utility to evaluate the integral. Graph the region whose area is given by the definite integral.

83. $\int_{0}^{4} \frac{x}{\sqrt{2x+1}}\,dx$

84. $\int_{0}^{2} x^3\sqrt{x+2}\,dx$

85. $\int_{3}^{7} x\sqrt{x-3}\,dx$

86. $\int_{1}^{5} x^2\sqrt{x-1}\,dx$

87. $\int_{0}^{3} \left(\theta + \cos\frac{\theta}{6}\right)d\theta$

88. $\int_{0}^{\pi/2} \sin 2x\,dx$

Writing **In Exercises 89 and 90, find the indefinite integral in two ways. Explain any difference in the forms of the answers.**

89. $\int (2x-1)^2\,dx$

90. $\int \sin x \cos x\,dx$

In Exercises 91–94, evaluate the integral using the properties of even and odd functions as an aid.

91. $\int_{-2}^{2} x^2(x^2+1)\,dx$

92. $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x\,dx$

93. $\int_{-2}^{2} x(x^2+1)^3\,dx$

94. $\int_{-\pi/2}^{\pi/2} \sin x \cos x\,dx$

95. Use $\int_0^2 x^2\,dx = \frac{8}{3}$ to evaluate the definite integrals without using the Fundamental Theorem of Calculus.

(a) $\int_{-2}^{0} x^2\,dx$

(b) $\int_{-2}^{2} x^2\,dx$

(c) $\int_{0}^{2} -x^2\,dx$

(d) $\int_{-2}^{0} 3x^2\,dx$