Before you begin the exercise set, be sure you realize that one of the most important steps in integration is *rewriting the integrand* in a form that fits the basic integration rules. To further illustrate this point, here are some additional examples.

Original Integral	*Rewrite*	*Integrate*	*Simplify*
$\int \frac{2}{\sqrt{x}}\,dx$	$2\int x^{-1/2}\,dx$	$2\left(\frac{x^{1/2}}{1/2}\right) + C$	$4x^{1/2} + C$
$\int (t^2 + 1)^2\,dt$	$\int (t^4 + 2t^2 + 1)\,dt$	$\frac{t^5}{5} + 2\left(\frac{t^3}{3}\right) + t + C$	$\frac{1}{5}t^5 + \frac{2}{3}t^3 + t + C$
$\int \frac{x^3 + 3}{x^2}\,dx$	$\int (x + 3x^{-2})\,dx$	$\frac{x^2}{2} + 3\left(\frac{x^{-1}}{-1}\right) + C$	$\frac{1}{2}x^2 - \frac{3}{x} + C$
$\int \sqrt[3]{x}(x - 4)\,dx$	$\int (x^{4/3} - 4x^{1/3})\,dx$	$\frac{x^{7/3}}{7/3} - 4\left(\frac{x^{4/3}}{4/3}\right) + C$	$\frac{3}{7}x^{4/3}(x - 7) + C$

EXERCISES FOR SECTION 4.1

Exercises 1–4, verify the statement by showing that the ivative of the right side equals the integrand of the left side.

$\int \left(-\frac{9}{x^4}\right) dx = \frac{3}{x^3} + C$

$\int \left(4x^3 - \frac{1}{x^2}\right) dx = x^4 + \frac{1}{x} + C$

$\int (x - 2)(x + 2)\,dx = \frac{1}{3}x^3 - 4x + C$

$\int \frac{x^2 - 1}{x^{3/2}}\,dx = \frac{2(x^2 + 3)}{3\sqrt{x}} + C$

Exercises 5–8, find the general solution of the differential ation and check the result by differentiation.

$\frac{dy}{dt} = 3t^2$

6. $\frac{dr}{d\theta} = \pi$

$\frac{dy}{dx} = x^{3/2}$

8. $\frac{dy}{dx} = 2x^{-3}$

Exercises 9–14, complete the table using Example 3 and the mples at the top of this page as a model.

Original Integral	*Rewrite*	*Integrate*	*Simplify*
$\int \sqrt[3]{x}\,dx$			
$\int \frac{1}{x^2}\,dx$			
$\int \frac{1}{x\sqrt{x}}\,dx$			
$\int x(x^2 + 3)\,dx$			
$\int \frac{1}{2x^3}\,dx$			
$\int \frac{1}{(3x)^2}\,dx$			

In Exercises 15–34, find the indefinite integral and check the result by differentiation.

15. $\int (x + 3)\,dx$

16. $\int (5 - x)\,dx$

17. $\int (2x - 3x^2)\,dx$

18. $\int (4x^3 + 6x^2 - 1)\,dx$

19. $\int (x^3 + 2)\,dx$

20. $\int (x^3 - 4x + 2)\,dx$

21. $\int (x^{3/2} + 2x + 1)\,dx$

22. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx$

23. $\int \sqrt[3]{x^2}\,dx$

24. $\int \left(\sqrt[4]{x^3} + 1\right) dx$

25. $\int \frac{1}{x^3}\,dx$

26. $\int \frac{1}{x^4}\,dx$

27. $\int \frac{x^2 + x + 1}{\sqrt{x}}\,dx$

28. $\int \frac{x^2 + 2x - 3}{x^4}\,dx$

29. $\int (x + 1)(3x - 2)\,dx$

30. $\int (2t^2 - 1)^2\,dt$

31. $\int y^2\sqrt{y}\,dy$

32. $\int (1 + 3t)t^2\,dt$

33. $\int dx$

34. $\int 3\,dt$

In Exercises 35–42, find the indefinite integral and check the result by differentiation.

35. $\int (2\sin x + 3\cos x)\,dx$

36. $\int (t^2 - \sin t)\,dt$

37. $\int (1 - \csc t \cot t)\,dt$

38. $\int (\theta^2 + \sec^2 \theta)\,d\theta$

39. $\int (\sec^2 \theta - \sin \theta)\,d\theta$

40. $\int \sec y(\tan y - \sec y)\,dy$

41. $\int (\tan^2 y + 1)\,dy$

42. $\int \frac{\cos x}{1 - \cos^2 x}\,dx$

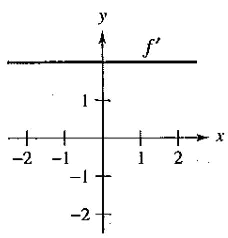
In Exercises 43 and 44, sketch the graphs of the function $g(x) = f(x) + C$ for $C = -2$, $C = 0$, and $C = 3$ on the same set of coordinate axes.

43. $f(x) = \cos x$

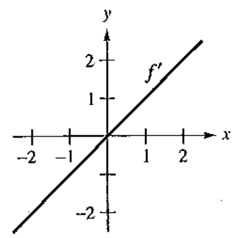
44. $f(x) = \sqrt{x}$

In Exercises 45–48, the graph of the derivative of a function is given. Sketch the graphs of *two* functions that have the given derivative. (There is more than one correct answer.) To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

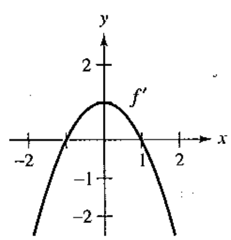
45.



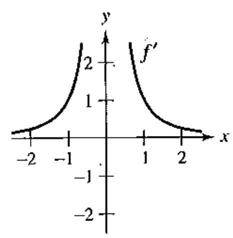
46.



47.

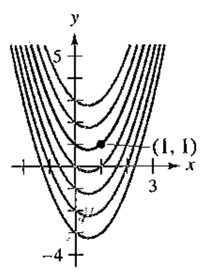


48.

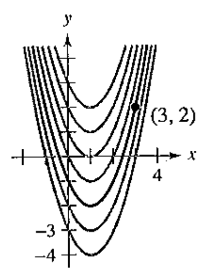


In Exercises 49–52, find the equation for *y*, given the derivative and the indicated point on the curve.

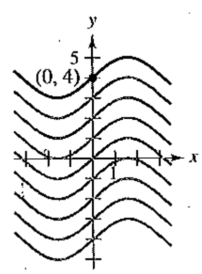
49. $\dfrac{dy}{dx} = 2x - 1$



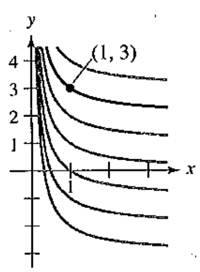
50. $\dfrac{dy}{dx} = 2(x - 1)$



51. $\dfrac{dy}{dx} = \cos x$

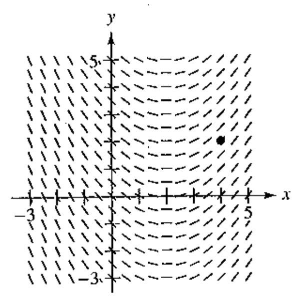


52. $\dfrac{dy}{dx} = -\dfrac{1}{x^2}, \quad x > 0$

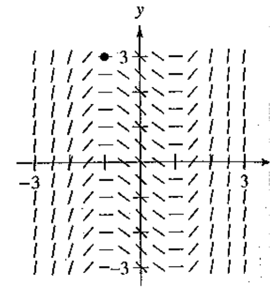


Slope Fields **In Exercises 53 and 54, a differential equatio point, and a slope field are given. A *slope field* (or *direction fi* consists of line segments with slopes given by the differen equation. These line segments give a visual perspective of slopes of the solutions of the differential equation. (a) Ske two approximate solutions of the differential equation on slope field, one of which passes through the indicated po (To print an enlarged copy of the graph, go to the web *www.mathgraphs.com*.) (b) Use integration to find the partic solution of the differential equation and use a graphing uti to graph the solution. Compare the result with the sketche part (a).**

53. $\dfrac{dy}{dx} = \dfrac{1}{2}x - 1, \quad (4, 2)$



54. $\dfrac{dy}{dx} = x^2 - 1, \quad (-1, 3)$



In Exercises 55–62, solve the differential equation.

55. $f'(x) = 4x, f(0) = 6$

56. $g'(x) = 6x^2, g(0) = -1$

57. $h'(t) = 8t^3 + 5, h(1) = -4$

58. $f'(s) = 6s - 8s^3, f(2) = 3$

59. $f''(x) = 2, \ f'(2) = 5, \ f(2) = 10$

60. $f''(x) = x^2, \ f'(0) = 6, \ f(0) = 3$

61. $f''(x) = x^{-3/2}, \ f'(4) = 2, \ f(0) = 0$

62. $f''(x) = \sin x, \ f'(0) = 1, \ f(0) = 6$

63. ***Tree Growth*** An evergreen nursery usually sells a ce shrub after 6 years of growth and shaping. The growth during those 6 years is approximated by

$$\frac{dh}{dt} = 1.5t + 5$$

where t is the time in years and h is the height in centime The seedlings are 12 centimeters tall when planted ($t = 0$)

(a) Find the height after t years.

(b) How tall are the shrubs when they are sold?

64. ***Population Growth*** The rate of growth dP/dt of a popul of bacteria is proportional to the square root of t, where P i population size and t is the time in days ($0 \le t \le 10$). Th

$$\frac{dP}{dt} = k\sqrt{t}.$$

The initial size of the population is 500. After 1 day the p lation has grown to 600. Estimate the population after 7 da