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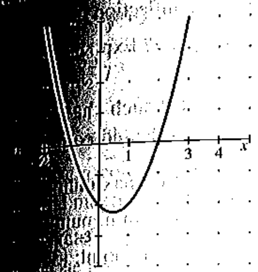
EXERCISES FOR SECTION 3.4

In Exercises 1–10, determine the open intervals on which the graph is concave upward or concave downward.

1. $y = x^2 - x - 2$



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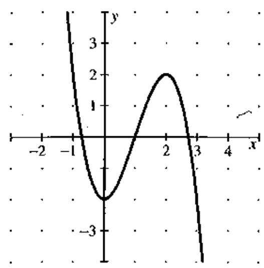
2. $y = -x^3 + 3x^2 - 2$



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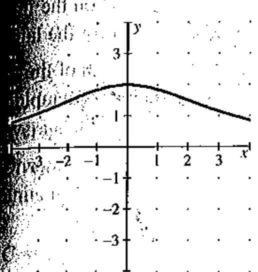
3. $f(x) = \dfrac{24}{x^2 + 12}$



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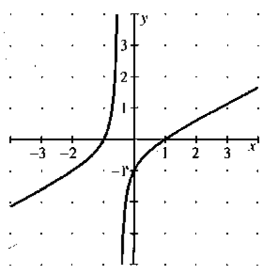
4. $f(x) = \dfrac{x^2 - 1}{2x + 1}$



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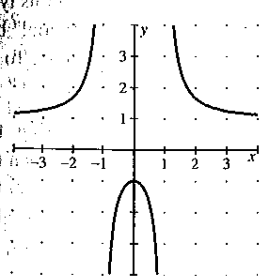
5. $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$



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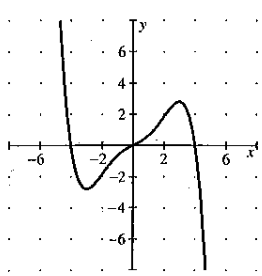
6. $y = \dfrac{-3x^5 + 40x^3 + 135x}{270}$



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7. $g(x) = 3x^2 - x^3$

8. $h(x) = x^5 - 5x + 2$

9. $y = 2x - \tan x, \quad \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

10. $y = x + \dfrac{2}{\sin x}, \quad (-\pi, \pi)$

In Exercises 11–26, find the points of inflection and discuss the concavity of the graph of the function.

11. $f(x) = x^3 - 6x^2 + 12x$
12. $f(x) = 2x^3 - 3x^2 - 12x + 5$
13. $f(x) = \frac{1}{4}x^4 - 2x^2$
14. $f(x) = 2x^4 - 8x + 3$
15. $f(x) = x(x - 4)^3$
16. $f(x) = x^3(x - 4)$
17. $f(x) = x\sqrt{x + 3}$
18. $f(x) = x\sqrt{x + 1}$
19. $f(x) = \dfrac{x}{x^2 + 1}$
20. $f(x) = \dfrac{x + 1}{\sqrt{x}}$
21. $f(x) = \sin \dfrac{x}{2}, \quad [0, 4\pi]$
22. $f(x) = 2 \csc \dfrac{3x}{2}, \quad (0, 2\pi)$
23. $f(x) = \sec\left(x - \dfrac{\pi}{2}\right), \quad (0, 4\pi)$
24. $f(x) = \sin x + \cos x, \quad [0, 2\pi]$
25. $f(x) = 2 \sin x + \sin 2x, \quad [0, 2\pi]$
26. $f(x) = x + 2 \cos x, \quad [0, 2\pi]$

In Exercises 27–40, find all relative extrema. Use the Second Derivative Test where applicable.

27. $f(x) = x^4 - 4x^3 + 2$
28. $f(x) = x^2 + 3x - 8$
29. $f(x) = (x - 5)^2$
30. $f(x) = -(x - 5)^2$
31. $f(x) = x^3 - 3x^2 + 3$
32. $f(x) = x^3 - 9x^2 + 27x$
33. $g(x) = x^2(6 - x)^3$
34. $g(x) = -\frac{1}{8}(x + 2)^2(x - 4)^2$
35. $f(x) = x^{2/3} - 3$
36. $f(x) = \sqrt{x^2 + 1}$
37. $f(x) = x + \dfrac{4}{x}$
38. $f(x) = \dfrac{x}{x - 1}$
39. $f(x) = \cos x - x, \quad [0, 4\pi]$
40. $f(x) = 2 \sin x + \cos 2x, \quad [0, 2\pi]$

In Exercises 41–44, use a computer algebra system to analyze the function over the indicated interval. (a) Find the first and second derivatives of the function. (b) Find any relative extrema and points of inflection. (c) Graph f, f', and f'' on the same set of coordinate axes and state the relationship between the behavior of f and the signs of f' and f''.

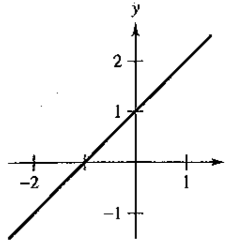
41. $f(x) = 0.2x^2(x - 3)^3, \quad [-1, 4]$
42. $f(x) = x^2\sqrt{6 - x^2}, \quad \left[-\sqrt{6}, \sqrt{6}\right]$
43. $f(x) = \sin x - \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x, \quad [0, \pi]$
44. $f(x) = \sqrt{2x}\sin x, \quad [0, 2\pi]$

Getting at the Concept

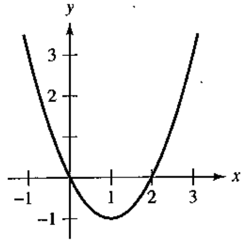
45. Consider a function f such that f' is increasing. Sketch graphs of f for (a) $f' < 0$ and (b) $f' > 0$.
46. Consider a function f such that f' is decreasing. Sketch graphs of f for (a) $f' < 0$ and (b) $f' > 0$.
47. Sketch the graph of a function f that does *not* have a point of inflection at $(c, f(c))$ even though $f''(c) = 0$.
48. S represents weekly sales of a product. What can be said of S' and S'' for each of the following?
    (a) The rate of change of sales is increasing.
    (b) Sales are increasing at a slower rate.
    (c) The rate of change of sales is constant.
    (d) Sales are steady.
    (e) Sales are declining, but at a slower rate.
    (f) Sales have bottomed out and have started to rise.

In Exercises 49–52, graph f, f', and f'' on the same set of coordinate axes. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

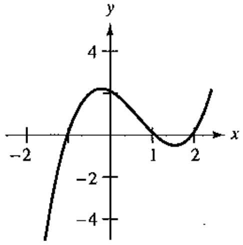
49. 41



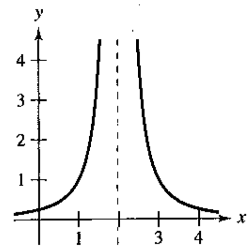
50. 42



51. 43



52. 44



Think About It **In Exercises 53–56, sketch the graph of a function f having the indicated characteristics.**

47 **53.** $f(2) = f(4) = 0$
$f(3)$ is defined.
$f'(x) < 0$ if $x < 3$
$f'(3)$ does not exist.
$f'(x) > 0$ if $x > 3$
$f''(x) < 0, x \neq 3$

48 **54.** $f(0) = f(2) = 0$
$f'(x) > 0$ if $x < 1$
$f'(1) = 0$
$f'(x) < 0$ if $x > 1$
$f''(x) < 0$

49 **55.** $f(2) = f(4) = 0$
$f'(x) > 0$ if $x < 3$
$f'(3)$ does not exist.
$f'(x) < 0$ if $x > 3$
$f''(x) > 0, x \neq 3$

50 **56.** $f(0) = f(2) = 0$
$f'(x) < 0$ if $x < 1$
$f'(1) = 0$
$f'(x) > 0$ if $x > 1$
$f''(x) > 0$

57. ***Think About It*** The figure shows the graph of f''. Sketch a graph of f. (The answer is not unique.) 53

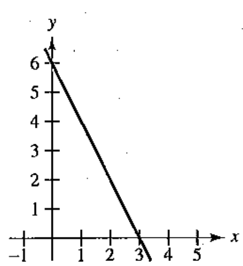


Figure for 57

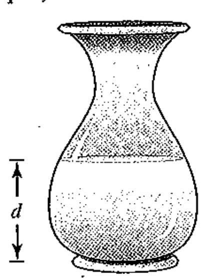


Figure for 58

58. ***Think About It*** Water is running into the vase shown in the figure at a constant rate. 54

(a) Graph the depth d of water in the vase as a function of time.

(b) Does the function have any extrema? Explain.

(c) Interpret the inflection points of the graph of d.

59. ***Conjecture*** Consider the function $f(x) = (x - 2)^n$. 55

(a) Use a graphing utility to graph f for $n = 1, 2, 3$, and 4. Use the graphs to make a conjecture about the relationship between n and any inflection points of the graph of f.

(b) Verify your conjecture in part (a).

56 **60.** (a) Graph $f(x) = \sqrt[3]{x}$ and identify the inflection point.

(b) Does $f''(x)$ exist at the inflection point? Explain.

In Exercises 61 and 62, find a, b, c, and d such that the cubic $f(x) = ax^3 + bx^2 + cx + d$ satisfies the indicated conditions.

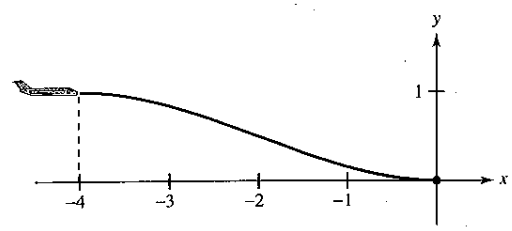
57 **61.** Relative maximum: $(3, 3)$
Relative minimum: $(5, 1)$
Inflection point: $(4, 2)$

52 **62.** Relative maximum: $(2, 4)$
Relative minimum: $(4, 2)$
Inflection point: $(3, 3)$

63. ***Aircraft Glide Path*** A small aircraft starts its descent from an altitude of 1 mile, 4 miles west of the runway (see figure). 59

(a) Find the cubic $f(x) = ax^3 + bx^2 + cx + d$ on the interval $[-4, 0]$ that describes a smooth glide path for the landing.

(b) The function in part (a) models the glide path of the plane. When would the plane be descending at the most rapid rate?



FOR FURTHER INFORMATION For more information on this type of modeling, see the article "How Not to Land at Lake Tahoe!" by Richard Barshinger in *The American Mathematical Monthly*. To view this article, go to the website *www.matharticles.com*.

64. ***Highway Design*** A section of highway connecting two hillsides with grades of 6% and 4% is to be built between two points that are separated by a horizontal distance of 2000 feet (see figure). At the point where the two hillsides come together, there is a 50-foot difference in elevation. 60

(a) Design a section of highway connecting the hillsides modeled by the function $f(x) = ax^3 + bx^2 + cx + d$ $(-1000 \leq x \leq 1000)$. At the points A and B, the slope of the model must match the grade of the hillside.

(b) Use a graphing utility to graph the model.

(c) Use a graphing utility to graph the derivative of the model.

(d) Determine the grade at the steepest part of the transitional section of the highway.