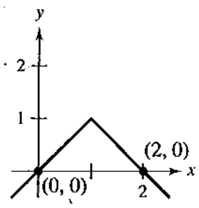
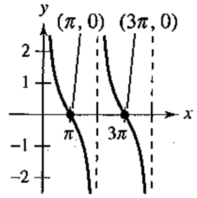
EXERCISES FOR SECTION 3.2

In Exercises 1 and 2, explain why Rolle's Theorem does not apply to the function even though there exist a and b such that $f(a) = f(b)$.

1. $f(x) = 1 - |x - 1|$



2. $f(x) = \cot \frac{x}{2}$



In Exercises 3–6, find the two x-intercepts of the function f and show that $f'(x) = 0$ at some point between the two intercepts.

3. $f(x) = x^2 - x - 2$
4. $f(x) = x(x - 3)$
5. $f(x) = x\sqrt{x + 4}$
6. $f(x) = -3x\sqrt{x + 1}$

In Exercises 7–20, determine whether Rolle's Theorem can be applied to f on the closed interval $[a, b]$. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = 0$.

7. $f(x) = x^2 - 2x, [0, 2]$
8. $f(x) = x^2 - 5x + 4, [1, 4]$
9. $f(x) = (x - 1)(x - 2)(x - 3), [1, 3]$
10. $f(x) = (x - 3)(x + 1)^2, [-1, 3]$
11. $f(x) = x^{2/3} - 1, [-8, 8]$
12. $f(x) = 3 - |x - 3|, [0, 6]$
13. $f(x) = \dfrac{x^2 - 2x - 3}{x + 2}, [-1, 3]$
14. $f(x) = \dfrac{x^2 - 1}{x}, [-1, 1]$
15. $f(x) = \sin x, [0, 2\pi]$
16. $f(x) = \cos x, [0, 2\pi]$
17. $f(x) = \dfrac{6x}{\pi} - 4\sin^2 x, \left[0, \dfrac{\pi}{6}\right]$
18. $f(x) = \cos 2x, \left[-\dfrac{\pi}{12}, \dfrac{\pi}{6}\right]$
19. $f(x) = \tan x, [0, \pi]$
20. $f(x) = \sec x, \left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

In Exercises 21–24, use a graphing utility to graph the function on the closed interval $[a, b]$. Determine whether Rolle's Theorem can be applied to f on the interval and, if so, find all values of c in the open interval (a, b) such that $f'(c) = 0$.

21. $f(x) = |x| - 1, [-1, 1]$
22. $f(x) = x - x^{1/3}, [0, 1]$
23. $f(x) = 4x - \tan \pi x, \left[-\frac{1}{4}, \frac{1}{4}\right]$
24. $f(x) = \dfrac{x}{2} - \sin \dfrac{\pi x}{6}, [-1, 0]$

25. ***Vertical Motion*** The height of a ball t seconds after it is thrown upward from a height of 32 feet and with an initial velocity of 48 feet per second is $f(t) = -16t^2 + 48t + 32$.

(a) Verify that $f(1) = f(2)$.

(b) According to Rolle's Theorem, what must be the velocity at some time in the interval $(1, 2)$? Find that time.

26. ***Reorder Costs*** The ordering and transportation cost C of components used in a manufacturing process is approximated by

$$C(x) = 10\left(\frac{1}{x} + \frac{x}{x + 3}\right)$$

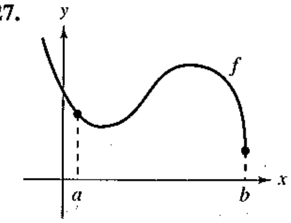
where C is measured in thousands of dollars and x is the order size in hundreds.

(a) Verify that $C(3) = C(6)$.

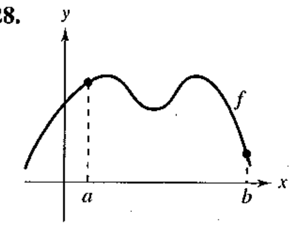
(b) According to Rolle's Theorem, the rate of change of cost must be 0 for some order size in the interval $(3, 6)$. Find that order size.

In Exercises 27 and 28, copy the graph and sketch the secant line to the graph through the points $(a, f(a))$ and $(b, f(b))$. Then sketch any tangent lines to the graph for each value of c guaranteed by the Mean Value Theorem. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

27.



28.



In Exercises 29 and 30, explain why the Mean Value Theorem does not apply to the function on the interval [0, 6].

29. $f(x) = \dfrac{1}{x - 3}$
30. $f(x) = |x - 3|$

In Exercises 31–38, determine whether the Mean Value Theorem can be applied to f on the closed interval $[a, b]$. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

31. $f(x) = x^2, [-2, 1]$
32. $f(x) = x(x^2 - x - 2), [-1, 1]$
33. $f(x) = x^{2/3}, [0, 1]$
34. $f(x) = \dfrac{x + 1}{x}, \left[\frac{1}{2}, 2\right]$
35. $f(x) = \sqrt{2 - x}, [-7, 2]$
36. $f(x) = x^3, [0, 1]$
37. $f(x) = \sin x, [0, \pi]$
38. $f(x) = 2\sin x + \sin 2x, [0, \pi]$

In Exercises 39–42, use a graphing utility to (a) graph the function f on the indicated interval, (b) find and graph the secant line through points on the graph of f at the endpoints of the indicated interval, and (c) find and graph any tangent lines to the graph of f that are parallel to the secant line.

39. $f(x) = \dfrac{x}{x + 1}, \left[-\frac{1}{2}, 2\right]$
40. $f(x) = x - 2\sin x, [-\pi, \pi]$
41. $f(x) = \sqrt{x}, [1, 9]$
42. $f(x) = -x^4 + 4x^3 + 8x^2 + 5, [0, 5]$