12.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

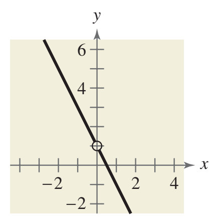
VOCABULARY: Fill in the blanks.

1. To evaluate the limit of a rational function that has common factors in its numerator and denominator, use the _______ _______ _______ .
2. The fraction $\frac{0}{0}$ has no meaning as a real number and therefore is called an _______ _______ .
3. The limit $\lim_{x \to c^-} f(x) = L_1$ is an example of a _______ _______ .
4. The limit of a _______ _______ is an expression of the form $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

SKILLS AND APPLICATIONS

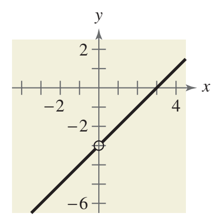
In Exercises 5–8, use the graph to determine each limit visually (if it exists). Then identify another function that agrees with the given function at all but one point.

5. $g(x) = \dfrac{-2x^2 + x}{x}$



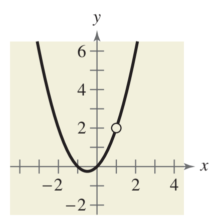
(a) $\lim_{x \to 0} g(x)$
(b) $\lim_{x \to -1} g(x)$
(c) $\lim_{x \to -2} g(x)$

6. $h(x) = \dfrac{x^2 - 3x}{x}$



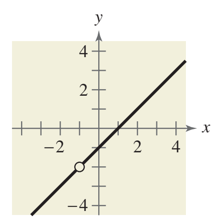
(a) $\lim_{x \to -2} h(x)$
(b) $\lim_{x \to 0} h(x)$
(c) $\lim_{x \to 3} h(x)$

7. $g(x) = \dfrac{x^3 - x}{x - 1}$



(a) $\lim_{x \to 1} g(x)$
(b) $\lim_{x \to -1} g(x)$
(c) $\lim_{x \to 0} g(x)$

8. $f(x) = \dfrac{x^2 - 1}{x + 1}$



(a) $\lim_{x \to 1} f(x)$
(b) $\lim_{x \to 2} f(x)$
(c) $\lim_{x \to -1} f(x)$

In Exercises 9–36, find the limit (if it exists). Use a graphing utility to verify your result graphically.

9. $\lim_{x \to 6} \dfrac{x - 6}{x^2 - 36}$

10. $\lim_{x \to 7} \dfrac{7 - x}{x^2 - 49}$

11. $\lim_{x \to 1} \dfrac{x^2 + 2x - 3}{x - 1}$

12. $\lim_{x \to -2} \dfrac{x^2 + 6x + 8}{x + 2}$

13. $\lim_{x \to -1} \dfrac{1 - 2x - 3x^2}{1 + x}$

14. $\lim_{x \to -3} \dfrac{2x^2 + 5x - 3}{x + 3}$

15. $\lim_{t \to 2} \dfrac{t^3 - 8}{t - 2}$

16. $\lim_{a \to -4} \dfrac{a^3 + 64}{a + 4}$

17. $\lim_{x \to 2} \dfrac{x^5 - 32}{x - 2}$

18. $\lim_{x \to 1} \dfrac{x^4 - 1}{x - 1}$

19. $\lim_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 3x + 2}$

20. $\lim_{x \to 4} \dfrac{x^2 - 2x - 8}{x^2 - 3x - 4}$

21. $\lim_{y \to 0} \dfrac{\sqrt{5 + y} - \sqrt{5}}{y}$

22. $\lim_{z \to 0} \dfrac{\sqrt{7 - z} - \sqrt{7}}{z}$

23. $\lim_{x \to 0} \dfrac{\sqrt{x + 3} - \sqrt{3}}{x}$

24. $\lim_{x \to 0} \dfrac{\sqrt{x + 4} - 2}{x}$

25. $\lim_{x \to 0} \dfrac{\sqrt{2x + 1} - 1}{x}$

26. $\lim_{x \to 9} \dfrac{3 - \sqrt{x}}{x - 9}$

27. $\lim_{x \to -3} \dfrac{\sqrt{x + 7} - 2}{x + 3}$

28. $\lim_{x \to 2} \dfrac{4 - \sqrt{18 - x}}{x - 2}$

29. $\lim_{x \to 0} \dfrac{\dfrac{1}{x + 1} - 1}{x}$

30. $\lim_{x \to 0} \dfrac{\dfrac{1}{x - 8} + \dfrac{1}{8}}{x}$

31. $\lim_{x \to 0} \dfrac{\dfrac{1}{x + 4} - \dfrac{1}{4}}{x}$

32. $\lim_{x \to 0} \dfrac{\dfrac{1}{2 + x} - \dfrac{1}{2}}{x}$

33. $\lim_{x \to 0} \dfrac{\sec x}{\tan x}$

34. $\lim_{x \to \pi} \dfrac{\csc x}{\cot x}$

35. $\lim_{x \to \pi/2} \dfrac{1 - \sin x}{\cos x}$

36. $\lim_{x \to \pi/2} \dfrac{\cos x - 1}{\sin x}$

In Exercises 37–48, use a graphing utility to graph the function and approximate the limit accurate to three decimal places.

37. $\lim_{x\to 0} \dfrac{e^{2x}-1}{x}$

38. $\lim_{x\to 0} \dfrac{1-e^{-x}}{x}$

39. $\lim_{x\to 0^+} (x \ln x)$

40. $\lim_{x\to 0^+} (x^2 \ln x)$

41. $\lim_{x\to 0} \dfrac{\sin 2x}{x}$

42. $\lim_{x\to 0} \dfrac{\sin 3x}{x}$

43. $\lim_{x\to 0} \dfrac{\tan x}{x}$

44. $\lim_{x\to 0} \dfrac{1-\cos 2x}{x}$

45. $\lim_{x\to 1} \dfrac{1-\sqrt[3]{x}}{1-x}$

46. $\lim_{x\to 1} \dfrac{\sqrt[3]{x}-x}{x-1}$

47. $\lim_{x\to 0} (1-x)^{2/x}$

48. $\lim_{x\to 0} (1+2x)^{1/x}$

GRAPHICAL, NUMERICAL, AND ALGEBRAIC ANALYSIS In Exercises 49–54, (a) graphically approximate the limit (if it exists) by using a graphing utility to graph the function, (b) numerically approximate the limit (if it exists) by using the *table* feature of a graphing utility to create a table, and (c) algebraically evaluate the limit (if it exists) by the appropriate technique(s).

49. $\lim_{x\to 1^-} \dfrac{x-1}{x^2-1}$

50. $\lim_{x\to 5^+} \dfrac{5-x}{25-x^2}$

51. $\lim_{x\to 2} \dfrac{x^4-1}{x^4-3x^2-4}$

52. $\lim_{x\to 2} \dfrac{x^4-2x^2-8}{x^4-6x^2+8}$

53. $\lim_{x\to 16^+} \dfrac{4-\sqrt{x}}{x-16}$

54. $\lim_{x\to 0^-} \dfrac{\sqrt{x+2}-\sqrt{2}}{x}$

In Exercises 55–62, graph the function. Determine the limit (if it exists) by evaluating the corresponding one-sided limits.

55. $\lim_{x\to 6} \dfrac{|x-6|}{x-6}$

56. $\lim_{x\to 2} \dfrac{|x-2|}{x-2}$

57. $\lim_{x\to 1} \dfrac{1}{x^2+1}$

58. $\lim_{x\to 1} \dfrac{1}{x^2-1}$

59. $\lim_{x\to 2} f(x)$ where $f(x) = \begin{cases} x-1, & x \le 2 \\ 2x-3, & x > 2 \end{cases}$

60. $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} 2x+1, & x < 1 \\ 4-x^2, & x \ge 1 \end{cases}$

61. $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} 4-x^2, & x \le 1 \\ 3-x, & x > 1 \end{cases}$

62. $\lim_{x\to 0} f(x)$ where $f(x) = \begin{cases} 4-x^2, & x \le 0 \\ x+4, & x > 0 \end{cases}$

In Exercises 63–68, use a graphing utility to graph the function and the equations $y = x$ and $y = -x$ in the same viewing window. Use the graph to find $\lim_{x\to 0} f(x)$.

63. $f(x) = x \cos x$

64. $f(x) = |x \sin x|$

65. $f(x) = |x| \sin x$

66. $f(x) = |x| \cos x$

67. $f(x) = x \sin \dfrac{1}{x}$

68. $f(x) = x \cos \dfrac{1}{x}$

In Exercises 69 and 70, state which limit can be evaluated by using direct substitution. Then evaluate or approximate each limit.

69. (a) $\lim_{x\to 0} x^2 \sin x^2$

(b) $\lim_{x\to 0} \dfrac{\sin x^2}{x^2}$

70. (a) $\lim_{x\to 0} \dfrac{x}{\cos x}$

(b) $\lim_{x\to 0} \dfrac{1-\cos x}{x}$

In Exercises 71–78, find $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

71. $f(x) = 2x + 1$

72. $f(x) = 3 - 4x$

73. $f(x) = \sqrt{x}$

74. $f(x) = \sqrt{x-2}$

75. $f(x) = x^2 - 3x$

76. $f(x) = 4 - 2x - x^2$

77. $f(x) = \dfrac{1}{x+2}$

78. $f(x) = \dfrac{1}{x-1}$

FREE-FALLING OBJECT In Exercises 79 and 80, use the position function

$s(t) = -16t^2 + 256$

which gives the height (in feet) of a free-falling object. The velocity at time $t = a$ seconds is given by $\lim_{t\to a} [s(a) - s(t)]/(a - t)$.

79. Find the velocity when $t = 1$ second.

80. Find the velocity when $t = 2$ seconds.

81. SALARY CONTRACT A union contract guarantees an 8% salary increase yearly for 3 years. For a current salary of \$30,000, the salaries $f(t)$ (in thousands of dollars) for the next 3 years are given by

$$f(t) = \begin{cases} 30.000, & 0 < t \le 1 \\ 32.400, & 1 < t \le 2 \\ 34.992, & 2 < t \le 3 \end{cases}$$

where t represents the time in years. Show that the limit of f as $t \to 2$ does not exist.

82. CONSUMER AWARENESS The cost of sending a package overnight is \$15 for the first pound and \$1.30 for each additional pound or portion of a pound. A plastic mailing bag can hold up to 3 pounds. The cost $f(x)$ of sending a package in a plastic mailing bag is given by

$$f(x) = \begin{cases} 15.00, & 0 < x \le 1 \\ 16.30, & 1 < x \le 2 \\ 17.60, & 2 < x \le 3 \end{cases}$$

where x represents the weight of the package (in pounds). Show that the limit of f as $x \to 1$ does not exist.

83. CONSUMER AWARENESS The cost of hooking up and towing a car is \$85 for the first mile and \$5 for each additional mile or portion of a mile. A model for the cost C (in dollars) is $C(x) = 85 - 5[\![-(x - 1)]\!]$, where x is the distance in miles. (Recall from Section 1.6 that $f(x) = [\![x]\!]$ = the greatest integer less than or equal to x.)

(a) Use a graphing utility to graph C for $0 < x \le 10$.

(b) Complete the table and observe the behavior of C as x approaches 5.5. Use the graph from part (a) and the table to find $\lim\limits_{x \to 5.5} C(x)$.

x	5	5.3	5.4	5.5	5.6	5.7	6
C				?			

(c) Complete the table and observe the behavior of C as x approaches 5. Does the limit of $C(x)$ as x approaches 5 exist? Explain.

x	4	4.5	4.9	5	5.1	5.5	6
C				?			

84. CONSUMER AWARENESS The cost C (in dollars) of making x photocopies at a copy shop is given by the function

$$C(x) = \begin{cases} 0.15x, & 0 < x \le 25 \\ 0.10x, & 25 < x \le 100 \\ 0.07x, & 100 < x \le 500 \\ 0.05x, & x > 500 \end{cases}.$$

(a) Sketch a graph of the function.

(b) Find each limit and interpret your result in the context of the situation.

(i) $\lim\limits_{x \to 15} C(x)$ (ii) $\lim\limits_{x \to 99} C(x)$ (iii) $\lim\limits_{x \to 305} C(x)$

(c) Create a table of values to show numerically that each limit does not exist.

(i) $\lim\limits_{x \to 25} C(x)$ (ii) $\lim\limits_{x \to 100} C(x)$ (iii) $\lim\limits_{x \to 500} C(x)$

(d) Explain how you can use the graph in part (a) to verify that the limits in part (c) do not exist.

EXPLORATION

TRUE OR FALSE? In Exercises 85 and 86, determine whether the statement is true or false. Justify your answer.

85. When your attempt to find the limit of a rational function yields the indeterminate form $\frac{0}{0}$, the rational function's numerator and denominator have a common factor.

86. If $f(c) = L$, then $\lim\limits_{x \to c} f(x) = L$.

87. THINK ABOUT IT

(a) Sketch the graph of a function for which $f(2)$ is defined but for which the limit of $f(x)$ as x approaches 2 does not exist.

(b) Sketch the graph of a function for which the limit of $f(x)$ as x approaches 1 is 4 but for which $f(1) \ne 4$.

88. CAPSTONE Given

$$f(x) = \begin{cases} 2x, & x \le 0 \\ x^2 + 1, & x > 0 \end{cases},$$

find each of the following limits. If the limit does not exist, explain why.

(a) $\lim\limits_{x \to 0^-} f(x)$ (b) $\lim\limits_{x \to 0^+} f(x)$ (c) $\lim\limits_{x \to 0} f(x)$

89. WRITING Consider the limit of the rational function given by $p(x)/q(x)$. What conclusion can you make if direct substitution produces each expression? Write a short paragraph explaining your reasoning.

(a) $\lim\limits_{x \to c} \dfrac{p(x)}{q(x)} = \dfrac{0}{1}$

(b) $\lim\limits_{x \to c} \dfrac{p(x)}{q(x)} = \dfrac{1}{1}$

(c) $\lim\limits_{x \to c} \dfrac{p(x)}{q(x)} = \dfrac{1}{0}$

(d) $\lim\limits_{x \to c} \dfrac{p(x)}{q(x)} = \dfrac{0}{0}$