10.9 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY

In Exercises 1–3, fill in the blanks.

1. The locus of a point in the plane that moves so that its distance from a fixed point (focus) is in a constant ratio to its distance from a fixed line (directrix) is a ________.

2. The constant ratio is the ________ of the conic and is denoted by ________.

3. An equation of the form $r = \dfrac{ep}{1 + e\cos\theta}$ has a ________ directrix to the ________ of the pole.

4. Match the conic with its eccentricity.

(a) $e < 1$ (b) $e = 1$ (c) $e > 1$

(i) parabola (ii) hyperbola (iii) ellipse

SKILLS AND APPLICATIONS

In Exercises 5–8, write the polar equation of the conic for $e = 1$, $e = 0.5$, and $e = 1.5$. Identify the conic for each equation. Verify your answers with a graphing utility.

5. $r = \dfrac{2e}{1 + e\cos\theta}$

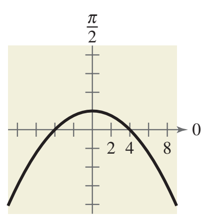
6. $r = \dfrac{2e}{1 - e\cos\theta}$

7. $r = \dfrac{2e}{1 - e\sin\theta}$

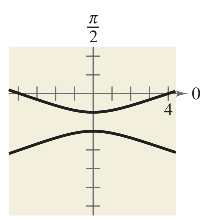
8. $r = \dfrac{2e}{1 + e\sin\theta}$

In Exercises 9–14, match the polar equation with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

(a)



(b)



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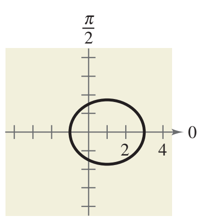

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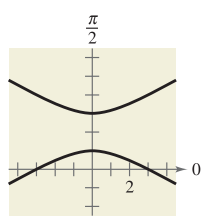

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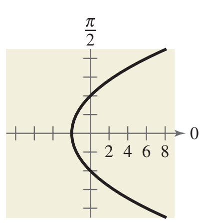
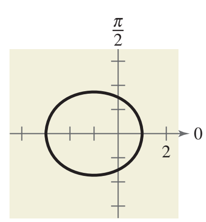


(f)



9. $r = \dfrac{4}{1 - \cos\theta}$

10. $r = \dfrac{3}{2 - \cos\theta}$

11. $r = \dfrac{3}{1 + 2\sin\theta}$

12. $r = \dfrac{3}{2 + \cos\theta}$

13. $r = \dfrac{4}{1 + \sin\theta}$

14. $r = \dfrac{4}{1 - 3\sin\theta}$

In Exercises 15–28, identify the conic and sketch its graph.

15. $r = \dfrac{3}{1 - \cos\theta}$

16. $r = \dfrac{7}{1 + \sin\theta}$

17. $r = \dfrac{5}{1 + \sin\theta}$

18. $r = \dfrac{6}{1 + \cos\theta}$

19. $r = \dfrac{2}{2 - \cos\theta}$

20. $r = \dfrac{4}{4 + \sin\theta}$

21. $r = \dfrac{6}{2 + \sin\theta}$

22. $r = \dfrac{9}{3 - 2\cos\theta}$

23. $r = \dfrac{3}{2 + 4\sin\theta}$

24. $r = \dfrac{5}{-1 + 2\cos\theta}$

25. $r = \dfrac{3}{2 - 6\cos\theta}$

26. $r = \dfrac{3}{2 + 6\sin\theta}$

27. $r = \dfrac{4}{2 - \cos\theta}$

28. $r = \dfrac{2}{2 + 3\sin\theta}$

In Exercises 29–34, use a graphing utility to graph the polar equation. Identify the graph.

29. $r = \dfrac{-1}{1 - \sin\theta}$

30. $r = \dfrac{-5}{2 + 4\sin\theta}$

31. $r = \dfrac{3}{-4 + 2\cos\theta}$

32. $r = \dfrac{4}{1 - 2\cos\theta}$

33. $r = \dfrac{14}{14 + 17\sin\theta}$

34. $r = \dfrac{12}{2 - \cos\theta}$

In Exercises 35–38, use a graphing utility to graph the rotated conic.

35. $r = \dfrac{3}{1 - \cos(\theta - \pi/4)}$ (See Exercise 15.)

36. $r = \dfrac{4}{4 + \sin(\theta - \pi/3)}$ (See Exercise 20.)

37. $r = \dfrac{6}{2 + \sin(\theta + \pi/6)}$ (See Exercise 21.)

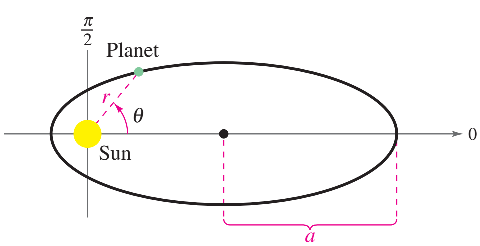
38. $r = \dfrac{5}{-1 + 2\cos(\theta + 2\pi/3)}$ (See Exercise 24.)

In Exercises 39–54, find a polar equation of the conic with its focus at the pole.

	Conic	*Eccentricity*	*Directrix*
39.	Parabola	$e = 1$	$x = -1$
40.	Parabola	$e = 1$	$y = -4$
41.	Ellipse	$e = \frac{1}{2}$	$y = 1$
42.	Ellipse	$e = \frac{3}{4}$	$y = -2$
43.	Hyperbola	$e = 2$	$x = 1$
44.	Hyperbola	$e = \frac{3}{2}$	$x = -1$

	Conic	*Vertex or Vertices*
45.	Parabola	$(1, -\pi/2)$
46.	Parabola	$(8, 0)$
47.	Parabola	$(5, \pi)$
48.	Parabola	$(10, \pi/2)$
49.	Ellipse	$(2, 0), (10, \pi)$
50.	Ellipse	$(2, \pi/2), (4, 3\pi/2)$
51.	Ellipse	$(20, 0), (4, \pi)$
52.	Hyperbola	$(2, 0), (8, 0)$
53.	Hyperbola	$(1, 3\pi/2), (9, 3\pi/2)$
54.	Hyperbola	$(4, \pi/2), (1, \pi/2)$

55. PLANETARY MOTION The planets travel in elliptical orbits with the sun at one focus. Assume that the focus is at the pole, the major axis lies on the polar axis, and the length of the major axis is $2a$ (see figure). Show that the polar equation of the orbit is $r = a(1 - e^2)/(1 - e\cos\theta)$, where e is the eccentricity.



56. PLANETARY MOTION Use the result of Exercise 55 to show that the minimum distance (*perihelion distance*) from the sun to the planet is $r = a(1 - e)$ and the maximum distance (*aphelion distance*) is $r = a(1 + e)$.

PLANETARY MOTION In Exercises 57–62, use the results of Exercises 55 and 56 to find the polar equation of the planet's orbit and the perihelion and aphelion distances.

57. Earth $a = 95.956 \times 10^6$ miles, $e = 0.0167$

58. Saturn $a = 1.427 \times 10^9$ kilometers, $e = 0.0542$

59. Venus $a = 108.209 \times 10^6$ kilometers, $e = 0.0068$

60. Mercury $a = 35.98 \times 10^6$ miles, $e = 0.2056$

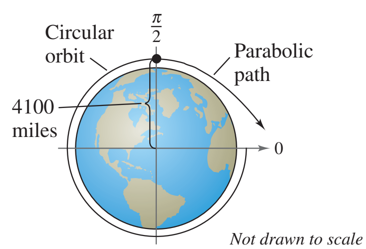
61. Mars $a = 141.63 \times 10^6$ miles, $e = 0.0934$

62. Jupiter $a = 778.41 \times 10^6$ kilometers, $e = 0.0484$

63. ASTRONOMY The comet Encke has an elliptical orbit with an eccentricity of $e \approx 0.847$. The length of the major axis of the orbit is approximately 4.42 astronomical units. Find a polar equation for the orbit. How close does the comet come to the sun?

64. ASTRONOMY The comet Hale-Bopp has an elliptical orbit with an eccentricity of $e \approx 0.995$. The length of the major axis of the orbit is approximately 500 astronomical units. Find a polar equation for the orbit. How close does the comet come to the sun?

65. SATELLITE TRACKING A satellite in a 100-mile-high circular orbit around Earth has a velocity of approximately 17,500 miles per hour. If this velocity is multiplied by $\sqrt{2}$, the satellite will have the minimum velocity necessary to escape Earth's gravity and will follow a parabolic path with the center of Earth as the focus (see figure).



(a) Find a polar equation of the parabolic path of the satellite (assume the radius of Earth is 4000 miles).

(b) Use a graphing utility to graph the equation you found in part (a).

(c) Find the distance between the surface of the Earth and the satellite when $\theta = 30°$.

(d) Find the distance between the surface of Earth and the satellite when $\theta = 60°$.

66. ROMAN COLISEUM The Roman Coliseum is an elliptical amphitheater measuring approximately 188 meters long and 156 meters wide.

(a) Find an equation to model the coliseum that is of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

(b) Find a polar equation to model the coliseum. (Assume $e \approx 0.5581$ and $p \approx 115.98$.)

(c) Use a graphing utility to graph the equations you found in parts (a) and (b). Are the graphs the same? Why or why not?

(d) In part (c), did you prefer graphing the rectangular equation or the polar equation? Explain.

EXPLORATION

TRUE OR FALSE? **In Exercises 67–70, determine whether the statement is true or false. Justify your answer.**

67. For a given value of $e > 1$ over the interval $\theta = 0$ to $\theta = 2\pi$, the graph of

$$r = \frac{ex}{1 - e\cos\theta}$$

is the same as the graph of

$$r = \frac{e(-x)}{1 + e\cos\theta}.$$

68. The graph of

$$r = \frac{4}{-3 - 3\sin\theta}$$

has a horizontal directrix above the pole.

69. The conic represented by the following equation is an ellipse.

$$r^2 = \frac{16}{9 - 4\cos\left(\theta + \dfrac{\pi}{4}\right)}$$

70. The conic represented by the following equation is a parabola.

$$r = \frac{6}{3 - 2\cos\theta}$$

71. WRITING Explain how the graph of each conic differs from the graph of $r = \dfrac{5}{1 + \sin\theta}$. (See Exercise 17.)

(a) $r = \dfrac{5}{1 - \cos\theta}$ (b) $r = \dfrac{5}{1 - \sin\theta}$

(c) $r = \dfrac{5}{1 + \cos\theta}$ (d) $r = \dfrac{5}{1 - \sin[\theta - (\pi/4)]}$

72. CAPSTONE In your own words, define the term *eccentricity* and explain how it can be used to classify conics.

73. Show that the polar equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{is} \quad r^2 = \frac{b^2}{1 - e^2\cos^2\theta}.$$

74. Show that the polar equation of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{is} \quad r^2 = \frac{-b^2}{1 - e^2\cos^2\theta}.$$

In Exercises 75–80, use the results of Exercises 73 and 74 to write the polar form of the equation of the conic.

75. $\dfrac{x^2}{169} + \dfrac{y^2}{144} = 1$ **76.** $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

77. $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$ **78.** $\dfrac{x^2}{36} - \dfrac{y^2}{4} = 1$

79. Hyperbola One focus: $\left(5, \dfrac{\pi}{2}\right)$

Vertices: $\left(4, \dfrac{\pi}{2}\right), \left(4, -\dfrac{\pi}{2}\right)$

80. Ellipse One focus: $(4, 0)$

Vertices: $(5, 0), (5, \pi)$

81. Consider the polar equation

$$r = \frac{4}{1 - 0.4\cos\theta}.$$

(a) Identify the conic without graphing the equation.

(b) Without graphing the following polar equations, describe how each differs from the given polar equation.

$$r_1 = \frac{4}{1 + 0.4\cos\theta} \qquad r_2 = \frac{4}{1 - 0.4\sin\theta}$$

(c) Use a graphing utility to verify your results in part (b).

82. The equation

$$r = \frac{ep}{1 \pm e\sin\theta}$$

is the equation of an ellipse with $e < 1$. What happens to the lengths of both the major axis and the minor axis when the value of e remains fixed and the value of p changes? Use an example to explain your reasoning.