10.8 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. The graph of $r = f(\sin\theta)$ is symmetric with respect to the line ________.

2. The graph of $r = g(\cos\theta)$ is symmetric with respect to the ________ ________.

3. The equation $r = 2 + \cos\theta$ represents a Text____ ________.

4. The equation $r = 2\cos\theta$ represents a ________.

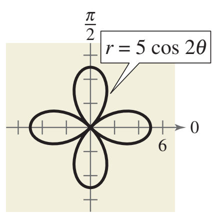
5. The equation $r^2 = 4\sin 2\theta$ represents a ________.

6. The equation $r = 1 + \sin\theta$ represents a ________.

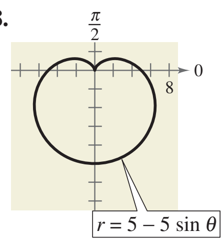
SKILLS AND APPLICATIONS

In Exercises 7–12, identify the type of polar graph.

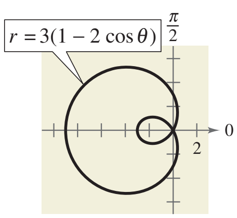
7.



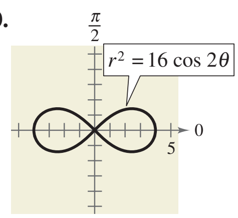
8.



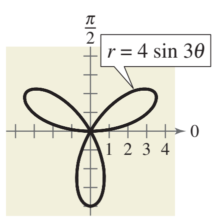
9.



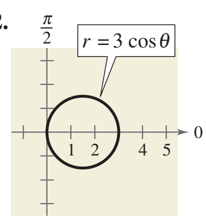
10.



11.



12.



In Exercises 13–18, test for symmetry with respect to $\theta = \pi/2$, the polar axis, and the pole.

13. $r = 4 + 3\cos\theta$

14. $r = 9\cos 3\theta$

15. $r = \dfrac{2}{1 + \sin\theta}$

16. $r = \dfrac{3}{2 + \cos\theta}$

17. $r^2 = 36\cos 2\theta$

18. $r^2 = 25\sin 2\theta$

In Exercises 19–22, find the maximum value of $|r|$ and any zeros of r.

19. $r = 10 - 10\sin\theta$

20. $r = 6 + 12\cos\theta$

21. $r = 4\cos 3\theta$

22. $r = 3\sin 2\theta$

In Exercises 23–48, sketch the graph of the polar equation using symmetry, zeros, maximum r-values, and any other additional points.

23. $r = 4$

24. $r = -7$

25. $r = \dfrac{\pi}{3}$

26. $r = -\dfrac{3\pi}{4}$

27. $r = \sin\theta$

28. $r = 4\cos\theta$

29. $r = 3(1 - \cos\theta)$

30. $r = 4(1 - \sin\theta)$

31. $r = 4(1 + \sin\theta)$

32. $r = 2(1 + \cos\theta)$

33. $r = 3 + 6\sin\theta$

34. $r = 4 - 3\sin\theta$

35. $r = 1 - 2\sin\theta$

36. $r = 2 - 4\cos\theta$

37. $r = 3 - 4\cos\theta$

38. $r = 4 + 3\cos\theta$

39. $r = 5\sin 2\theta$

40. $r = 2\cos 2\theta$

41. $r = 6\cos 3\theta$

42. $r = 3\sin 3\theta$

43. $r = 2\sec\theta$

44. $r = 5\csc\theta$

45. $r = \dfrac{3}{\sin\theta - 2\cos\theta}$

46. $r = \dfrac{6}{2\sin\theta - 3\cos\theta}$

47. $r^2 = 9\cos 2\theta$

48. $r^2 = 4\sin\theta$

In Exercises 49–58, use a graphing utility to graph the polar equation. Describe your viewing window.

49. $r = \dfrac{9}{4}$

50. $r = -\dfrac{5}{2}$

51. $r = \dfrac{5\pi}{8}$

52. $r = -\dfrac{\pi}{10}$

53. $r = 8\cos\theta$

54. $r = \cos 2\theta$

55. $r = 3(2 - \sin\theta)$

56. $r = 2\cos(3\theta - 2)$

57. $r = 8\sin\theta\cos^2\theta$

58. $r = 2\csc\theta + 5$

In Exercises 59–64, use a graphing utility to graph the polar equation. Find an interval for θ for which the graph is traced *only once*.

59. $r = 3 - 8\cos\theta$

60. $r = 5 + 4\cos\theta$

61. $r = 2\cos\left(\dfrac{3\theta}{2}\right)$

62. $r = 3\sin\left(\dfrac{5\theta}{2}\right)$

63. $r^2 = 16 \sin 2\theta$ **64.** $r^2 = \dfrac{1}{\theta}$

In Exercises 65–68, use a graphing utility to graph the polar equation and show that the indicated line is an asymptote of the graph.

	Name of Graph	*Polar Equation*	*Asymptote*
65.	Conchoid	$r = 2 - \sec\theta$	$x = -1$
66.	Conchoid	$r = 2 + \csc\theta$	$y = 1$
67.	Hyperbolic spiral	$r = \dfrac{3}{\theta}$	$y = 3$
68.	Strophoid	$r = 2\cos 2\theta \sec\theta$	$x = -2$

EXPLORATION

TRUE OR FALSE? In Exercises 69 and 70, determine whether the statement is true or false. Justify your answer.

69. In the polar coordinate system, if a graph that has symmetry with respect to the polar axis were folded on the line $\theta = 0$, the portion of the graph above the polar axis would coincide with the portion of the graph below the polar axis.

70. In the polar coordinate system, if a graph that has symmetry with respect to the pole were folded on the line $\theta = 3\pi/4$, the portion of the graph on one side of the fold would coincide with the portion of the graph on the other side of the fold.

71. Sketch the graph of $r = 6\cos\theta$ over each interval. Describe the part of the graph obtained in each case.

(a) $0 \le \theta \le \dfrac{\pi}{2}$ (b) $\dfrac{\pi}{2} \le \theta \le \pi$

(c) $-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$ (d) $\dfrac{\pi}{4} \le \theta \le \dfrac{3\pi}{4}$

72. GRAPHICAL REASONING Use a graphing utility to graph the polar equation $r = 6[1 + \cos(\theta - \phi)]$ for (a) $\phi = 0$, (b) $\phi = \pi/4$, and (c) $\phi = \pi/2$. Use the graphs to describe the effect of the angle ϕ. Write the equation as a function of $\sin\theta$ for part (c).

73. The graph of $r = f(\theta)$ is rotated about the pole through an angle ϕ. Show that the equation of the rotated graph is $r = f(\theta - \phi)$.

74. Consider the graph of $r = f(\sin\theta)$.

(a) Show that if the graph is rotated counterclockwise $\pi/2$ radians about the pole, the equation of the rotated graph is $r = f(-\cos\theta)$.

(b) Show that if the graph is rotated counterclockwise π radians about the pole, the equation of the rotated graph is $r = f(-\sin\theta)$.

(c) Show that if the graph is rotated counterclockwise $3\pi/2$ radians about the pole, the equation of the rotated graph is $r = f(\cos\theta)$.

In Exercises 75–78, use the results of Exercises 73 and 74.

75. Write an equation for the limaçon $r = 2 - \sin\theta$ after it has been rotated through the given angle.

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) π (d) $\dfrac{3\pi}{2}$

76. Write an equation for the rose curve $r = 2\sin 2\theta$ after it has been rotated through the given angle.

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{2\pi}{3}$ (d) π

77. Sketch the graph of each equation.

(a) $r = 1 - \sin\theta$ (b) $r = 1 - \sin\left(\theta - \dfrac{\pi}{4}\right)$

78. Sketch the graph of each equation.

(a) $r = 3\sec\theta$ (b) $r = 3\sec\left(\theta - \dfrac{\pi}{4}\right)$

(c) $r = 3\sec\left(\theta + \dfrac{\pi}{3}\right)$ (d) $r = 3\sec\left(\theta - \dfrac{\pi}{2}\right)$

79. THINK ABOUT IT How many petals do the rose curves given by $r = 2\cos 4\theta$ and $r = 2\sin 3\theta$ have? Determine the numbers of petals for the curves given by $r = 2\cos n\theta$ and $r = 2\sin n\theta$, where n is a positive integer.

80. Use a graphing utility to graph and identify $r = 2 + k\sin\theta$ for $k = 0, 1, 2,$ and 3.

81. Consider the equation $r = 3\sin k\theta$.

(a) Use a graphing utility to graph the equation for $k = 1.5$. Find the interval for θ over which the graph is traced only once.

(b) Use a graphing utility to graph the equation for $k = 2.5$. Find the interval for θ over which the graph is traced only once.

(c) Is it possible to find an interval for θ over which the graph is traced only once for any rational number k? Explain.

82. CAPSTONE Write a brief paragraph that describes why some polar curves have equations that are simpler in polar form than in rectangular form. Besides a circle, give an example of a curve that is simpler in polar form than in rectangular form. Give an example of a curve that is simpler in rectangular form than in polar form.