10.7 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. The origin of the polar coordinate system is called the ________.
2. For the point (r, θ), r is the ________ ________ from O to P and θ is the ________ ________ , counterclockwise from the polar axis to the line segment $\overline{OP}$.
3. To plot the point (r, θ), use the ________ coordinate system.
4. The polar coordinates (r, θ) are related to the rectangular coordinates (x, y) as follows:
   $x =$ ________ $y =$ ________ $\tan\theta =$ ________ $r^2 =$ ________

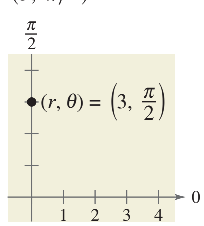
SKILLS AND APPLICATIONS

In Exercises 5–18, plot the point given in polar coordinates and find two additional polar representations of the point, using $-2\pi < \theta < 2\pi$.

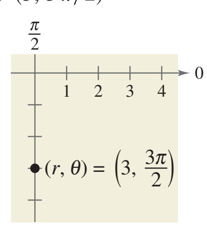
5. $\left(2, \frac{5\pi}{6}\right)$
6. $\left(3, \frac{5\pi}{4}\right)$
7. $\left(4, -\frac{\pi}{3}\right)$
8. $\left(-1, -\frac{3\pi}{4}\right)$
9. $(2, 3\pi)$
10. $\left(4, \frac{5\pi}{2}\right)$
11. $\left(-2, \frac{2\pi}{3}\right)$
12. $\left(-3, \frac{11\pi}{6}\right)$
13. $\left(0, -\frac{7\pi}{6}\right)$
14. $\left(0, -\frac{7\pi}{2}\right)$
15. $\left(\sqrt{2}, 2.36\right)$
16. $\left(2\sqrt{2}, 4.71\right)$
17. $(-3, -1.57)$
18. $(-5, -2.36)$

In Exercises 19–28, a point in polar coordinates is given. Convert the point to rectangular coordinates.

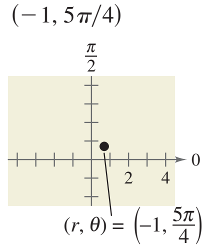
19. $(3, \pi/2)$



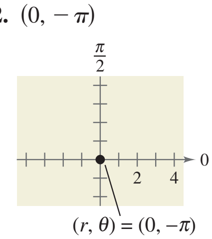
20. $(3, 3\pi/2)$



21. $(-1, 5\pi/4)$



22. $(0, -\pi)$



23. $(2, 3\pi/4)$
24. $(1, 5\pi/4)$
25. $(-2, 7\pi/6)$
26. $(-3, 5\pi/6)$
27. $(-2.5, 1.1)$
28. $(-2, 5.76)$

In Exercises 29–36, use a graphing utility to find the rectangular coordinates of the point given in polar coordinates. Round your results to two decimal places.

29. $(2, 2\pi/9)$
30. $(4, 11\pi/9)$
31. $(-4.5, 1.3)$
32. $(8.25, 3.5)$
33. $(2.5, 1.58)$
34. $(5.4, 2.85)$
35. $(-4.1, -0.5)$
36. $(8.2, -3.2)$

In Exercises 37–54, a point in rectangular coordinates is given. Convert the point to polar coordinates.

37. $(1, 1)$
38. $(2, 2)$
39. $(-3, -3)$
40. $(-4, -4)$
41. $(-6, 0)$
42. $(3, 0)$
43. $(0, -5)$
44. $(0, 5)$
45. $(-3, 4)$
46. $(-4, -3)$
47. $\left(-\sqrt{3}, -\sqrt{3}\right)$
48. $\left(-\sqrt{3}, \sqrt{3}\right)$
49. $\left(\sqrt{3}, -1\right)$
50. $\left(-1, \sqrt{3}\right)$
51. $(6, 9)$
52. $(6, 2)$
53. $(5, 12)$
54. $(7, 15)$

In Exercises 55–64, use a graphing utility to find one set of polar coordinates for the point given in rectangular coordinates.

55. $(3, -2)$
56. $(-4, -2)$
57. $(-5, 2)$
58. $(7, -2)$
59. $\left(\sqrt{3}, 2\right)$
60. $\left(5, -\sqrt{2}\right)$
61. $\left(\frac{5}{2}, \frac{4}{3}\right)$
62. $\left(\frac{9}{5}, \frac{11}{2}\right)$
63. $\left(\frac{7}{4}, \frac{3}{2}\right)$
64. $\left(-\frac{7}{9}, -\frac{3}{4}\right)$

In Exercises 65–84, convert the rectangular equation to polar form. Assume $a > 0$.

65. $x^2 + y^2 = 9$
66. $x^2 + y^2 = 16$

67. $y = 4$ **68.** $y = x$
69. $x = 10$ **70.** $x = 4a$
71. $y = -2$ **72.** $y = 1$
73. $3x - y + 2 = 0$ **74.** $3x + 5y - 2 = 0$
75. $xy = 16$ **76.** $2xy = 1$
77. $y^2 - 8x - 16 = 0$ **78.** $(x^2 + y^2)^2 = 9(x^2 - y^2)$
79. $x^2 + y^2 = a^2$ **80.** $x^2 + y^2 = 9a^2$
81. $x^2 + y^2 - 2ax = 0$ **82.** $x^2 + y^2 - 2ay = 0$
83. $y^3 = x^2$ **84.** $y^2 = x^3$

In Exercises 85–108, convert the polar equation to rectangular form.

85. $r = 4 \sin \theta$ **86.** $r = 2 \cos \theta$
87. $r = -2 \cos \theta$ **88.** $r = -5 \sin \theta$
89. $\theta = 2\pi/3$ **90.** $\theta = 5\pi/3$
91. $\theta = 11\pi/6$ **92.** $\theta = 5\pi/6$
93. $r = 4$ **94.** $r = 10$
95. $r = 4 \csc \theta$ **96.** $r = 2 \csc \theta$
97. $r = -3 \sec \theta$ **98.** $r = -\sec \theta$
99. $r^2 = \cos \theta$ **100.** $r^2 = 2 \sin \theta$
101. $r^2 = \sin 2\theta$ **102.** $r^2 = \cos 2\theta$
103. $r = 2 \sin 3\theta$ **104.** $r = 3 \cos 2\theta$
105. $r = \dfrac{2}{1 + \sin \theta}$ **106.** $r = \dfrac{1}{1 - \cos \theta}$
107. $r = \dfrac{6}{2 - 3 \sin \theta}$ **108.** $r = \dfrac{6}{2 \cos \theta - 3 \sin \theta}$

In Exercises 109–118, describe the graph of the polar equation and find the corresponding rectangular equation. Sketch its graph.

109. $r = 6$ **110.** $r = 8$
111. $\theta = \pi/6$ **112.** $\theta = 3\pi/4$
113. $r = 2 \sin \theta$ **114.** $r = 4 \cos \theta$
115. $r = -6 \cos \theta$ **116.** $r = -3 \sin \theta$
117. $r = 3 \sec \theta$ **118.** $r = 2 \csc \theta$

EXPLORATION

TRUE OR FALSE? In Exercises 119 and 120, determine whether the statement is true or false. Justify your answer.

119. If $\theta_1 = \theta_2 + 2\pi n$ for some integer n, then (r, θ_1) and (r, θ_2) represent the same point on the polar coordinate system.

120. If $|r_1| = |r_2|$, then (r_1, θ) and (r_2, θ) represent the same point on the polar coordinate system.

121. Convert the polar equation $r = 2(h \cos \theta + k \sin \theta)$ to rectangular form and verify that it is the equation of a circle. Find the radius of the circle and the rectangular coordinates of the center of the circle.

122. Convert the polar equation $r = \cos \theta + 3 \sin \theta$ to rectangular form and identify the graph.

123. THINK ABOUT IT

(a) Show that the distance between the points (r_1, θ_1) and (r_2, θ_2) is $\sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_1 - \theta_2)}$.

(b) Describe the positions of the points relative to each other for $\theta_1 = \theta_2$. Simplify the Distance Formula for this case. Is the simplification what you expected? Explain.

(c) Simplify the Distance Formula for $\theta_1 - \theta_2 = 90°$. Is the simplification what you expected? Explain.

(d) Choose two points on the polar coordinate system and find the distance between them. Then choose different polar representations of the same two points and apply the Distance Formula again. Discuss the result.

124. GRAPHICAL REASONING

(a) Set the window format of your graphing utility on rectangular coordinates and locate the cursor at any position off the coordinate axes. Move the cursor horizontally and observe any changes in the displayed coordinates of the points. Explain the changes in the coordinates. Now repeat the process moving the cursor vertically.

(b) Set the window format of your graphing utility on polar coordinates and locate the cursor at any position off the coordinate axes. Move the cursor horizontally and observe any changes in the displayed coordinates of the points. Explain the changes in the coordinates. Now repeat the process moving the cursor vertically.

(c) Explain why the results of parts (a) and (b) are not the same.

125. GRAPHICAL REASONING

(a) Use a graphing utility in *polar* mode to graph the equation $r = 3$.

(b) Use the *trace* feature to move the cursor around the circle. Can you locate the point $(3, 5\pi/4)$?

(c) Can you find other polar representations of the point $(3, 5\pi/4)$? If so, explain how you did it.

126. CAPSTONE In the rectangular coordinate system, each point (x, y) has a unique representation. Explain why this is not true for a point (r, θ) in the polar coordinate system.