10.6 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. If f and g are continuous functions of t on an interval I, the set of ordered pairs $(f(t), g(t))$ is a ________ ________ C.
2. The ________ of a curve is the direction in which the curve is traced out for increasing values of the parameter.
3. The process of converting a set of parametric equations to a corresponding rectangular equation is called ________ the ________.
4. A curve traced by a point on the circumference of a circle as the circle rolls along a straight line in a plane is called a ________.

SKILLS AND APPLICATIONS

5. Consider the parametric equations $x = \sqrt{t}$ and $y = 3 - t$.
   (a) Create a table of x- and y-values using $t = 0, 1, 2, 3$, and 4.
   (b) Plot the points (x, y) generated in part (a), and sketch a graph of the parametric equations.
   (c) Find the rectangular equation by eliminating the parameter. Sketch its graph. How do the graphs differ?
6. Consider the parametric equations $x = 4\cos^2\theta$ and $y = 2\sin\theta$.
   (a) Create a table of x- and y-values using $\theta = -\pi/2, -\pi/4, 0, \pi/4$, and $\pi/2$.
   (b) Plot the points (x, y) generated in part (a), and sketch a graph of the parametric equations.
   (c) Find the rectangular equation by eliminating the parameter. Sketch its graph. How do the graphs differ?

In Exercises 7–26, (a) sketch the curve represented by the parametric equations (indicate the orientation of the curve) and (b) eliminate the parameter and write the corresponding rectangular equation whose graph represents the curve. Adjust the domain of the resulting rectangular equation if necessary.

7. $x = t - 1$, $y = 3t + 1$
8. $x = 3 - 2t$, $y = 2 + 3t$
9. $x = \frac{1}{4}t$, $y = t^2$
10. $x = t$, $y = t^3$
11. $x = t + 2$, $y = t^2$
12. $x = \sqrt{t}$, $y = 1 - t$
13. $x = t + 1$, $y = \dfrac{t}{t+1}$
14. $x = t - 1$, $y = \dfrac{t}{t-1}$
15. $x = 2(t + 1)$, $y = |t - 2|$
16. $x = |t - 1|$, $y = t + 2$
17. $x = 4\cos\theta$, $y = 2\sin\theta$
18. $x = 2\cos\theta$, $y = 3\sin\theta$
19. $x = 6\sin 2\theta$, $y = 6\cos 2\theta$
20. $x = \cos\theta$, $y = 2\sin 2\theta$
21. $x = 1 + \cos\theta$, $y = 1 + 2\sin\theta$
22. $x = 2 + 5\cos\theta$, $y = -6 + 4\sin\theta$
23. $x = e^{-t}$, $y = e^{3t}$
24. $x = e^{2t}$, $y = e^t$
25. $x = t^3$, $y = 3\ln t$
26. $x = \ln 2t$, $y = 2t^2$

In Exercises 27 and 28, determine how the plane curves differ from each other.

27. (a) $x = t$, $y = 2t + 1$ (b) $x = \cos\theta$, $y = 2\cos\theta + 1$
    (c) $x = e^{-t}$, $y = 2e^{-t} + 1$ (d) $x = e^t$, $y = 2e^t + 1$
28. (a) $x = t$, $y = t^2 - 1$ (b) $x = t^2$, $y = t^4 - 1$
    (c) $x = \sin t$, $y = \sin^2 t - 1$ (d) $x = e^t$, $y = e^{2t} - 1$

In Exercises 29–32, eliminate the parameter and obtain the standard form of the rectangular equation.

29. Line through (x_1, y_1) and (x_2, y_2):
    $x = x_1 + t(x_2 - x_1),\ y = y_1 + t(y_2 - y_1)$
30. Circle: $x = h + r\cos\theta,\ y = k + r\sin\theta$
31. Ellipse: $x = h + a\cos\theta,\ y = k + b\sin\theta$
32. Hyperbola: $x = h + a\sec\theta,\ y = k + b\tan\theta$

In Exercises 33–40, use the results of Exercises 29–32 to find a set of parametric equations for the line or conic.

33. Line: passes through $(0, 0)$ and $(3, 6)$
34. Line: passes through $(3, 2)$ and $(-6, 3)$
35. Circle: center: $(3, 2)$; radius: 4
36. Circle: center: $(5, -3)$; radius: 4

37. Ellipse: vertices: $(\pm 5, 0)$; foci: $(\pm 4, 0)$

38. Ellipse: vertices: $(3, 7)$, $(3, -1)$; foci: $(3, 5)$, $(3, 1)$

39. Hyperbola: vertices: $(\pm 4, 0)$; foci: $(\pm 5, 0)$

40. Hyperbola: vertices: $(\pm 2, 0)$; foci: $(\pm 4, 0)$

In Exercises 41–48, find a set of parametric equations for the rectangular equation using (a) $t = x$ and (b) $t = 2 - x$.

41. $y = 3x - 2$

42. $x = 3y - 2$

43. $y = 2 - x$

44. $y = x^2 + 1$

45. $y = x^2 - 3$

46. $y = 1 - 2x^2$

47. $y = \dfrac{1}{x}$

48. $y = \dfrac{1}{2x}$

In Exercises 49–56, use a graphing utility to graph the curve represented by the parametric equations.

49. Cycloid: $x = 4(\theta - \sin\theta)$, $y = 4(1 - \cos\theta)$

50. Cycloid: $x = \theta + \sin\theta$, $y = 1 - \cos\theta$

51. Prolate cycloid: $x = \theta - \frac{3}{2}\sin\theta$, $y = 1 - \frac{3}{2}\cos\theta$

52. Prolate cycloid: $x = 2\theta - 4\sin\theta$, $y = 2 - 4\cos\theta$

53. Hypocycloid: $x = 3\cos^3\theta$, $y = 3\sin^3\theta$

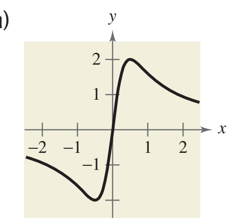
54. Curtate cycloid: $x = 8\theta - 4\sin\theta$, $y = 8 - 4\cos\theta$

55. Witch of Agnesi: $x = 2\cot\theta$, $y = 2\sin^2\theta$

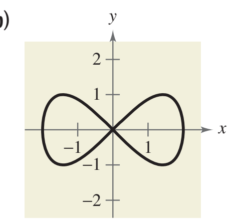
56. Folium of Descartes: $x = \dfrac{3t}{1 + t^3}$, $y = \dfrac{3t^2}{1 + t^3}$

In Exercises 57–60, match the parametric equations with the correct graph and describe the domain and range. [The graphs are labeled (a), (b), (c), and (d).]

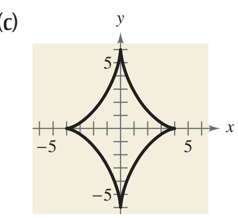
(a)



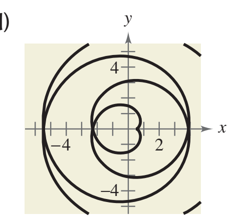
(b)



(c)



(d)



57. Lissajous curve: $x = 2\cos\theta$, $y = \sin 2\theta$

58. Evolute of ellipse: $x = 4\cos^3\theta$, $y = 6\sin^3\theta$

59. Involute of circle: $x = \frac{1}{2}(\cos\theta + \theta\sin\theta)$

$y = \frac{1}{2}(\sin\theta - \theta\cos\theta)$

60. Serpentine curve: $x = \frac{1}{2}\cot\theta$, $y = 4\sin\theta\cos\theta$

PROJECTILE MOTION A projectile is launched at a height of h feet above the ground at an angle of θ with the horizontal. The initial velocity is v_0 feet per second, and the path of the projectile is modeled by the parametric equations

$$x = (v_0 \cos\theta)t \quad \text{and} \quad y = h + (v_0 \sin\theta)t - 16t^2.$$

In Exercises 61 and 62, use a graphing utility to graph the paths of a projectile launched from ground level at each value of θ and v_0. For each case, use the graph to approximate the maximum height and the range of the projectile.

61. (a) $\theta = 60°$, $v_0 = 88$ feet per second

(b) $\theta = 60°$, $v_0 = 132$ feet per second

(c) $\theta = 45°$, $v_0 = 88$ feet per second

(d) $\theta = 45°$, $v_0 = 132$ feet per second

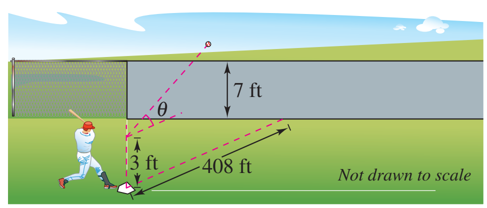
62. (a) $\theta = 15°$, $v_0 = 50$ feet per second

(b) $\theta = 15°$, $v_0 = 120$ feet per second

(c) $\theta = 10°$, $v_0 = 50$ feet per second

(d) $\theta = 10°$, $v_0 = 120$ feet per second

63. SPORTS The center field fence in Yankee Stadium is 7 feet high and 408 feet from home plate. A baseball is hit at a point 3 feet above the ground. It leaves the bat at an angle of θ degrees with the horizontal at a speed of 100 miles per hour (see figure).



(a) Write a set of parametric equations that model the path of the baseball.

(b) Use a graphing utility to graph the path of the baseball when $\theta = 15°$. Is the hit a home run?

(c) Use the graphing utility to graph the path of the baseball when $\theta = 23°$. Is the hit a home run?

(d) Find the minimum angle required for the hit to be a home run.

64. **SPORTS** An archer releases an arrow from a bow at a point 5 feet above the ground. The arrow leaves the bow at an angle of 15° with the horizontal and at an initial speed of 225 feet per second.
   (a) Write a set of parametric equations that model the path of the arrow.
   (b) Assuming the ground is level, find the distance the arrow travels before it hits the ground. (Ignore air resistance.)
   (c) Use a graphing utility to graph the path of the arrow and approximate its maximum height.
   (d) Find the total time the arrow is in the air.

65. **PROJECTILE MOTION** Eliminate the parameter t from the parametric equations

$$x = (v_0 \cos \theta)t \quad \text{and} \quad y = h + (v_0 \sin \theta)t - 16t^2$$

for the motion of a projectile to show that the rectangular equation is

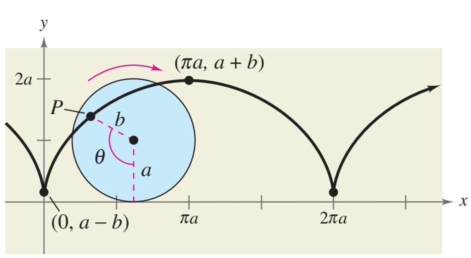
$$y = -\frac{16 \sec^2 \theta}{v_0^2}x^2 + (\tan \theta)x + h.$$

66. **PATH OF A PROJECTILE** The path of a projectile is given by the rectangular equation

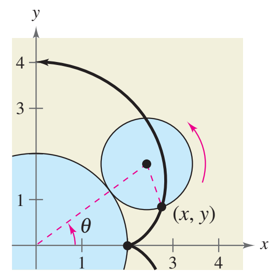
$$y = 7 + x - 0.02x^2.$$

   (a) Use the result of Exercise 65 to find h, v_0, and θ. Find the parametric equations of the path.
   (b) Use a graphing utility to graph the rectangular equation for the path of the projectile. Confirm your answer in part (a) by sketching the curve represented by the parametric equations.
   (c) Use the graphing utility to approximate the maximum height of the projectile and its range.

67. **CURTATE CYCLOID** A wheel of radius a units rolls along a straight line without slipping. The curve traced by a point P that is b units from the center ($b < a$) is called a **curtate cycloid** (see figure). Use the angle θ shown in the figure to find a set of parametric equations for the curve.



68. **EPICYCLOID** A circle of radius one unit rolls around the outside of a circle of radius two units without slipping. The curve traced by a point on the circumference of the smaller circle is called an **epicycloid** (see figure). Use the angle θ shown in the figure to find a set of parametric equations for the curve.



EXPLORATION

TRUE OR FALSE? In Exercises 69 and 70, determine whether the statement is true or false. Justify your answer.

69. The two sets of parametric equations $x = t$, $y = t^2 + 1$ and $x = 3t$, $y = 9t^2 + 1$ have the same rectangular equation.

70. If y is a function of t, and x is a function of t, then y must be a function of x.

71. **WRITING** Write a short paragraph explaining why parametric equations are useful.

72. **WRITING** Explain the process of sketching a plane curve given by parametric equations. What is meant by the orientation of the curve?

73. Use a graphing utility set in *parametric* mode to enter the parametric equations from Example 2. Over what values should you let t vary to obtain the graph shown in Figure 10.55?

74. **CAPSTONE** Consider the parametric equations $x = 8 \cos t$ and $y = 8 \sin t$.
   (a) Describe the curve represented by the parametric equations.
   (b) How does the curve represented by the parametric equations $x = 8 \cos t + 3$ and $y = 8 \sin t + 6$ compare with the curve described in part (a)?
   (c) How does the original curve change when cosine and sine are interchanged?