Systems of Quadratic Equations

To find the points of intersection of two conics, you can use elimination or substitution, as demonstrated in Examples 5 and 6.

EXAMPLE 5 Solving a Quadratic System

Solve the system of quadratic equations.

$$\begin{cases} x^2 + y^2 - 16x + 39 = 0 & \text{Equation 1} \\ x^2 - y^2 - 9 = 0 & \text{Equation 2} \end{cases}$$

Algebraic Solution

You can eliminate the y^2-term by adding the two equations. The resulting equation can then be solved for x.

$$2x^2 - 16x + 30 = 0$$

$$2(x - 3)(x - 5) = 0$$

There are two real solutions: $x = 3$ and $x = 5$. The corresponding y-values are $y = 0$ and $y = \pm 4$. So, the graphs have three points of intersection:

$$(3, 0), (5, 4), \text{ and } (5, -4).$$

Graphical Solution

Begin by solving each equation for y as follows.

$$y = \pm\sqrt{-x^2 + 16x - 39} \qquad y = \pm\sqrt{x^2 - 9}$$

Use a graphing utility to graph all four equations $y_1 = \sqrt{-x^2 + 16x - 39}$, $y_2 = -\sqrt{-x^2 + 16x - 39}$, $y_3 = \sqrt{x^2 - 9}$, and $y_4 = -\sqrt{x^2 - 9}$ in the same viewing window. In Figure 10.41, you can see that the graphs appear to intersect at the points $(3, 0)$, $(5, 4)$, and $(5, -4)$. Use the *intersect* feature to confirm this.

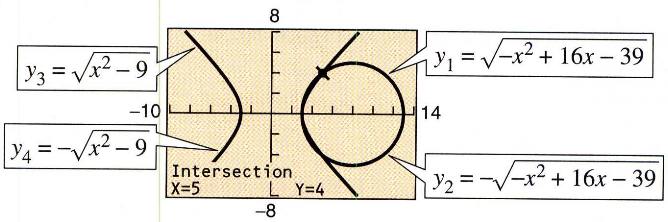

Figure 10.41

EXAMPLE 6 Solving a Quadratic System by Substitution

Solve the system of quadratic equations.

$$\begin{cases} x^2 + 4y^2 - 4x - 8y + 4 = 0 & \text{Equation 1} \\ x^2 + 4y - 4 = 0 & \text{Equation 2} \end{cases}$$

Solution

Because Equation 2 has no y^2-term, solve the equation for y to obtain $y = 1 - (1/4)x^2$. Next, substitute this into Equation 1 and solve for x.

$$x^2 + 4\left(1 - \frac{1}{4}x^2\right)^2 - 4x - 8\left(1 - \frac{1}{4}x^2\right) + 4 = 0$$

$$x^2 + 4 - 2x^2 + \frac{1}{4}x^4 - 4x - 8 + 2x^2 + 4 = 0$$

$$x^4 + 4x^2 - 16x = 0$$

$$x(x - 2)(x^2 + 2x + 8) = 0$$

In factored form, you can see that the equation has two real solutions: $x = 0$ and $x = 2$. The corresponding values of y are $y = 1$ and $y = 0$. This implies that the solutions of the system of equations are $(0, 1)$ and $(2, 0)$, as shown in Figure 10.42.

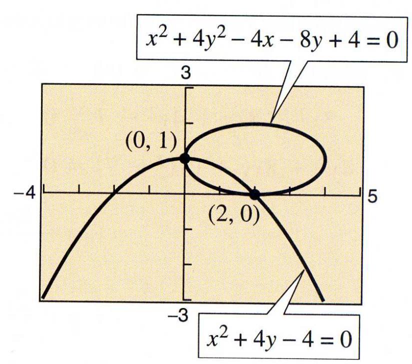

Figure 10.42