10.4 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. A ________ is the set of all points (x, y) in a plane, the difference of whose distances from two distinct fixed points, called ________, is a positive constant.

2. The graph of a hyperbola has two disconnected parts called ________.

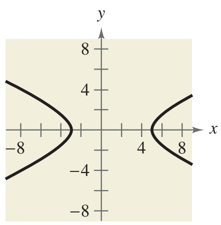
3. The line segment connecting the vertices of a hyperbola is called the ________ ________, and the midpoint of the line segment is the ________ of the hyperbola.

4. Each hyperbola has two ________ that intersect at the center of the hyperbola.

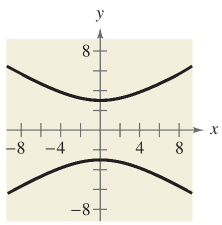
SKILLS AND APPLICATIONS

In Exercises 5–8, match the equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]

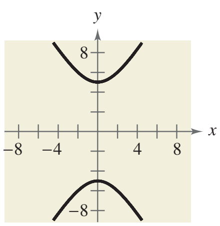
(a)



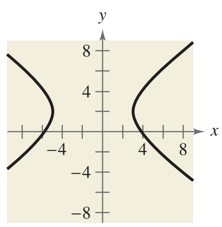
(b)



(c)



(d)



5. $\dfrac{y^2}{9} - \dfrac{x^2}{25} = 1$

6. $\dfrac{y^2}{25} - \dfrac{x^2}{9} = 1$

7. $\dfrac{(x-1)^2}{16} - \dfrac{y^2}{4} = 1$

8. $\dfrac{(x+1)^2}{16} - \dfrac{(y-2)^2}{9} = 1$

In Exercises 9–22, find the center, vertices, foci, and the equations of the asymptotes of the hyperbola, and sketch its graph using the asymptotes as an aid.

9. $x^2 - y^2 = 1$

10. $\dfrac{x^2}{9} - \dfrac{y^2}{25} = 1$

11. $\dfrac{y^2}{25} - \dfrac{x^2}{81} = 1$

12. $\dfrac{x^2}{36} - \dfrac{y^2}{4} = 1$

13. $\dfrac{y^2}{1} - \dfrac{x^2}{4} = 1$

14. $\dfrac{y^2}{9} - \dfrac{x^2}{1} = 1$

15. $\dfrac{(x-1)^2}{4} - \dfrac{(y+2)^2}{1} = 1$

16. $\dfrac{(x+3)^2}{144} - \dfrac{(y-2)^2}{25} = 1$

17. $\dfrac{(y+6)^2}{1/9} - \dfrac{(x-2)^2}{1/4} = 1$

18. $\dfrac{(y-1)^2}{1/4} - \dfrac{(x+3)^2}{1/16} = 1$

19. $9x^2 - y^2 - 36x - 6y + 18 = 0$

20. $x^2 - 9y^2 + 36y - 72 = 0$

21. $x^2 - 9y^2 + 2x - 54y - 80 = 0$

22. $16y^2 - x^2 + 2x + 64y + 63 = 0$

In Exercises 23–28, find the center, vertices, foci, and the equations of the asymptotes of the hyperbola. Use a graphing utility to graph the hyperbola and its asymptotes.

23. $2x^2 - 3y^2 = 6$

24. $6y^2 - 3x^2 = 18$

25. $4x^2 - 9y^2 = 36$

26. $25x^2 - 4y^2 = 100$

27. $9y^2 - x^2 + 2x + 54y + 62 = 0$

28. $9x^2 - y^2 + 54x + 10y + 55 = 0$

In Exercises 29–34, find the standard form of the equation of the hyperbola with the given characteristics and center at the origin.

29. Vertices: $(0, \pm 2)$; foci: $(0, \pm 4)$

30. Vertices: $(\pm 4, 0)$; foci: $(\pm 6, 0)$

31. Vertices: $(\pm 1, 0)$; asymptotes: $y = \pm 5x$

32. Vertices: $(0, \pm 3)$; asymptotes: $y = \pm 3x$

33. Foci: $(0, \pm 8)$; asymptotes: $y = \pm 4x$

34. Foci: $(\pm 10, 0)$; asymptotes: $y = \pm \frac{3}{4}x$

In Exercises 35–46, find the standard form of the equation of the hyperbola with the given characteristics.

35. Vertices: $(2, 0), (6, 0)$; foci: $(0, 0), (8, 0)$

36. Vertices: $(2, 3), (2, -3)$; foci: $(2, 6), (2, -6)$

37. Vertices: $(4, 1), (4, 9)$; foci: $(4, 0), (4, 10)$

38. Vertices: $(-2, 1), (2, 1)$; foci: $(-3, 1), (3, 1)$

39. Vertices: $(2, 3), (2, -3)$;
passes through the point $(0, 5)$

40. Vertices: $(-2, 1), (2, 1)$;
passes through the point $(5, 4)$

41. Vertices: $(0, 4), (0, 0)$;
passes through the point $(\sqrt{5}, -1)$

42. Vertices: $(1, 2), (1, -2)$;
passes through the point $(0, \sqrt{5})$

43. Vertices: $(1, 2), (3, 2)$;
asymptotes: $y = x,\ y = 4 - x$

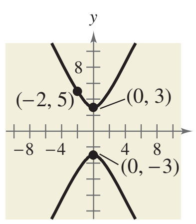
44. Vertices: $(3, 0), (3, 6)$;
asymptotes: $y = 6 - x,\ y = x$

45. Vertices: $(0, 2), (6, 2)$;
asymptotes: $y = \frac{2}{3}x,\ y = 4 - \frac{2}{3}x$

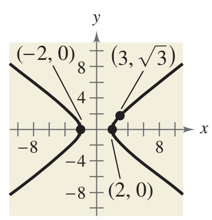
46. Vertices: $(3, 0), (3, 4)$;
asymptotes: $y = \frac{2}{3}x,\ y = 4 - \frac{2}{3}x$

In Exercises 47–50, write the standard form of the equation of the hyperbola.

47.



48.



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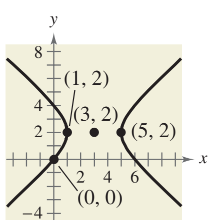
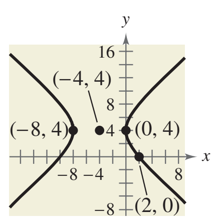
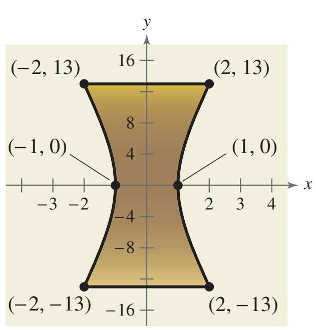


50.



51. ART A sculpture has a hyperbolic cross section (see figure).



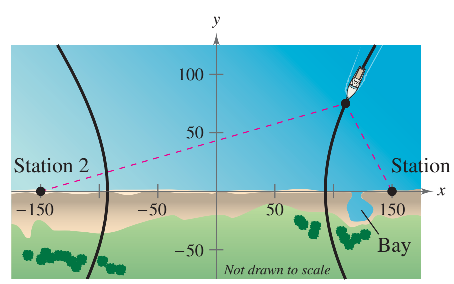
(a) Write an equation that models the curved sides of the sculpture.

(b) Each unit in the coordinate plane represents 1 foot. Find the width of the sculpture at a height of 5 feet.

52. SOUND LOCATION You and a friend live 4 miles apart (on the same "east-west" street) and are talking on the phone. You hear a clap of thunder from lightning in a storm, and 18 seconds later your friend hears the thunder. Find an equation that gives the possible places where the lightning could have occurred. (Assume that the coordinate system is measured in feet and that sound travels at 1100 feet per second.)

53. SOUND LOCATION Three listening stations located at $(3300, 0)$, $(3300, 1100)$, and $(-3300, 0)$ monitor an explosion. The last two stations detect the explosion 1 second and 4 seconds after the first, respectively. Determine the coordinates of the explosion. (Assume that the coordinate system is measured in feet and that sound travels at 1100 feet per second.)

54. LORAN Long distance radio navigation for aircraft and ships uses synchronized pulses transmitted by widely separated transmitting stations. These pulses travel at the speed of light (186,000 miles per second). The difference in the times of arrival of these pulses at an aircraft or ship is constant on a hyperbola having the transmitting stations as foci. Assume that two stations, 300 miles apart, are positioned on the rectangular coordinate system at points with coordinates $(-150, 0)$ and $(150, 0)$, and that a ship is traveling on a hyperbolic path with coordinates $(x, 75)$ (see figure).



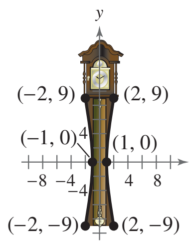
(a) Find the x-coordinate of the position of the ship if the time difference between the pulses from the transmitting stations is 1000 microseconds (0.001 second).

(b) Determine the distance between the ship and station 1 when the ship reaches the shore.

(c) The ship wants to enter a bay located between the two stations. The bay is 30 miles from station 1. What should be the time difference between the pulses?

(d) The ship is 60 miles offshore when the time difference in part (c) is obtained. What is the position of the ship?

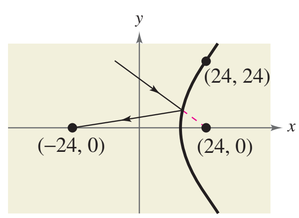
55. PENDULUM The base for a pendulum of a clock has the shape of a hyperbola (see figure).



(a) Write an equation of the cross section of the base.

(b) Each unit in the coordinate plane represents $\frac{1}{2}$ foot. Find the width of the base of the pendulum 4 inches from the bottom.

56. HYPERBOLIC MIRROR A hyperbolic mirror (used in some telescopes) has the property that a light ray directed at a focus will be reflected to the other focus. The focus of a hyperbolic mirror (see figure) has coordinates (24, 0). Find the vertex of the mirror if the mount at the top edge of the mirror has coordinates (24, 24).



In Exercises 57–72, classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola.

57. $9x^2 + 4y^2 - 18x + 16y - 119 = 0$

58. $x^2 + y^2 - 4x - 6y - 23 = 0$

59. $4x^2 - y^2 - 4x - 3 = 0$

60. $y^2 - 6y - 4x + 21 = 0$

61. $y^2 - 4x^2 + 4x - 2y - 4 = 0$

62. $x^2 + y^2 - 4x + 6y - 3 = 0$

63. $y^2 + 12x + 4y + 28 = 0$

64. $4x^2 + 25y^2 + 16x + 250y + 541 = 0$

65. $4x^2 + 3y^2 + 8x - 24y + 51 = 0$

66. $4y^2 - 2x^2 - 4y - 8x - 15 = 0$

67. $25x^2 - 10x - 200y - 119 = 0$

68. $4y^2 + 4x^2 - 24x + 35 = 0$

69. $x^2 - 6x - 2y + 7 = 0$

70. $9x^2 + 4y^2 - 90x + 8y + 228 = 0$

71. $100x^2 + 100y^2 - 100x + 400y + 409 = 0$

72. $4x^2 - y^2 + 4x + 2y - 1 = 0$

EXPLORATION

TRUE OR FALSE? In Exercises 73–76, determine whether the statement is true or false. Justify your answer.

73. In the standard form of the equation of a hyperbola, the larger the ratio of b to a, the larger the eccentricity of the hyperbola.

74. In the standard form of the equation of a hyperbola, the trivial solution of two intersecting lines occurs when $b = 0$.

75. If $D \neq 0$ and $E \neq 0$, then the graph of $x^2 - y^2 + Dx + Ey = 0$ is a hyperbola.

76. If the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a, b > 0$, intersect at right angles, then $a = b$.

77. Consider a hyperbola centered at the origin with a horizontal transverse axis. Use the definition of a hyperbola to derive its standard form.

78. WRITING Explain how the central rectangle of a hyperbola can be used to sketch its asymptotes.

79. THINK ABOUT IT Change the equation of the hyperbola so that its graph is the bottom half of the hyperbola.

$$9x^2 - 54x - 4y^2 + 8y + 41 = 0$$

80. CAPSTONE Given the hyperbolas

$$\frac{x^2}{16} - \frac{y^2}{9} = 1 \quad \text{and} \quad \frac{y^2}{9} - \frac{x^2}{16} = 1$$

describe any common characteristics that the hyperbolas share, as well as any differences in the graphs of the hyperbolas. Verify your results by using a graphing utility to graph each of the hyperbolas in the same viewing window.

81. A circle and a parabola can have 0, 1, 2, 3, or 4 points of intersection. Sketch the circle given by $x^2 + y^2 = 4$. Discuss how this circle could intersect a parabola with an equation of the form $y = x^2 + C$. Then find the values of C for each of the five cases described below. Use a graphing utility to verify your results.

(a) No points of intersection

(b) One point of intersection

(c) Two points of intersection

(d) Three points of intersection

(e) Four points of intersection