10.3 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. An ________ is the set of all points (x, y) in a plane, the sum of whose distances from two distinct fixed points, called ________, is constant.

2. The chord joining the vertices of an ellipse is called the ________ ________, and its midpoint is the ________ of the ellipse.

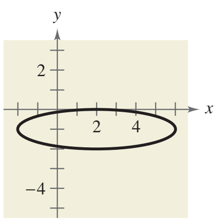
3. The chord perpendicular to the major axis at the center of the ellipse is called the ________ ________ of the ellipse.

4. The concept of ________ is used to measure the ovalness of an ellipse.

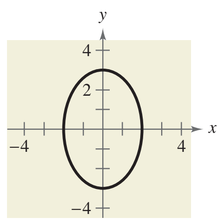
SKILLS AND APPLICATIONS

In Exercises 5–10, match the equation with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

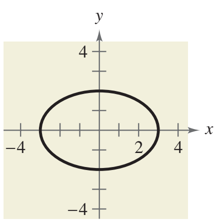
(a)



(b)



(c)



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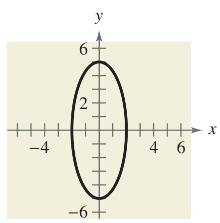
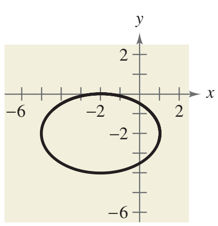
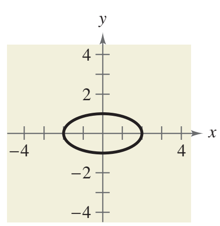


(e)



(f)



5. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

6. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

7. $\dfrac{x^2}{4} + \dfrac{y^2}{25} = 1$

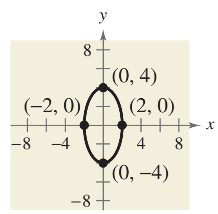
8. $\dfrac{x^2}{4} + y^2 = 1$

9. $\dfrac{(x-2)^2}{16} + (y+1)^2 = 1$

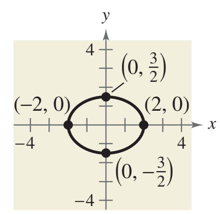
10. $\dfrac{(x+2)^2}{9} + \dfrac{(y+2)^2}{4} = 1$

In Exercises 11–18, find the standard form of the equation of the ellipse with the given characteristics and center at the origin.

11.



12.



13. Vertices: $(\pm 7, 0)$; foci: $(\pm 2, 0)$

14. Vertices: $(0, \pm 8)$; foci: $(0, \pm 4)$

15. Foci: $(\pm 5, 0)$; major axis of length 14

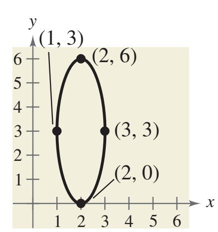
16. Foci: $(\pm 2, 0)$; major axis of length 10

17. Vertices: $(0, \pm 5)$; passes through the point $(4, 2)$

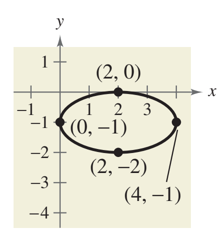
18. Vertical major axis; passes through the points $(0, 6)$ and $(3, 0)$

In Exercises 19–28, find the standard form of the equation of the ellipse with the given characteristics.

19.



20.



21. Vertices: $(0, 2)$, $(8, 2)$; minor axis of length 2

22. Foci: $(0, 0)$, $(4, 0)$; major axis of length 6

23. Foci: $(0, 0)$, $(0, 8)$; major axis of length 16

24. Center: $(2, -1)$; vertex: $\left(2, \frac{1}{2}\right)$; minor axis of length 2

25. Center: $(0, 4)$; $a = 2c$; vertices: $(-4, 4)$, $(4, 4)$

26. Center: $(3, 2)$; $a = 3c$; foci: $(1, 2)$, $(5, 2)$

27. Vertices: (0, 2), (4, 2); endpoints of the minor axis: (2, 3), (2, 1)

28. Vertices: (5, 0), (5, 12); endpoints of the minor axis: (1, 6), (9, 6)

In Exercises 29–52, identify the conic as a circle or an ellipse. Then find the center, radius, vertices, foci, and eccentricity of the conic (if applicable), and sketch its graph.

29. $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

30. $\dfrac{x^2}{16} + \dfrac{y^2}{81} = 1$

31. $\dfrac{x^2}{25} + \dfrac{y^2}{25} = 1$

32. $\dfrac{x^2}{9} + \dfrac{y^2}{9} = 1$

33. $\dfrac{x^2}{5} + \dfrac{y^2}{9} = 1$

34. $\dfrac{x^2}{64} + \dfrac{y^2}{28} = 1$

35. $\dfrac{(x-4)^2}{16} + \dfrac{(y+1)^2}{25} = 1$

36. $\dfrac{(x+3)^2}{12} + \dfrac{(y-2)^2}{16} = 1$

37. $\dfrac{x^2}{4/9} + \dfrac{(y+1)^2}{4/9} = 1$

38. $\dfrac{(x+5)^2}{9/4} + (y-1)^2 = 1$

39. $(x+2)^2 + \dfrac{(y+4)^2}{1/4} = 1$

40. $\dfrac{(x-3)^2}{25/4} + \dfrac{(y-1)^2}{25/4} = 1$

41. $9x^2 + 4y^2 + 36x - 24y + 36 = 0$

42. $9x^2 + 4y^2 - 54x + 40y + 37 = 0$

43. $x^2 + y^2 - 2x + 4y - 31 = 0$

44. $x^2 + 5y^2 - 8x - 30y - 39 = 0$

45. $3x^2 + y^2 + 18x - 2y - 8 = 0$

46. $6x^2 + 2y^2 + 18x - 10y + 2 = 0$

47. $x^2 + 4y^2 - 6x + 20y - 2 = 0$

48. $x^2 + y^2 - 4x + 6y - 3 = 0$

49. $9x^2 + 9y^2 + 18x - 18y + 14 = 0$

50. $16x^2 + 25y^2 - 32x + 50y + 16 = 0$

51. $9x^2 + 25y^2 - 36x - 50y + 60 = 0$

52. $16x^2 + 16y^2 - 64x + 32y + 55 = 0$

In Exercises 53–56, use a graphing utility to graph the ellipse. Find the center, foci, and vertices. (Recall that it may be necessary to solve the equation for *y* and obtain two equations.)

53. $5x^2 + 3y^2 = 15$

54. $3x^2 + 4y^2 = 12$

55. $12x^2 + 20y^2 - 12x + 40y - 37 = 0$

56. $36x^2 + 9y^2 + 48x - 36y - 72 = 0$

In Exercises 57–60, find the eccentricity of the ellipse.

57. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

58. $\dfrac{x^2}{25} + \dfrac{y^2}{36} = 1$

59. $x^2 + 9y^2 - 10x + 36y + 52 = 0$

60. $4x^2 + 3y^2 - 8x + 18y + 19 = 0$

61. Find an equation of the ellipse with vertices $(\pm 5, 0)$ and eccentricity $e = \frac{3}{5}$.

62. Find an equation of the ellipse with vertices $(0, \pm 8)$ and eccentricity $e = \frac{1}{2}$.

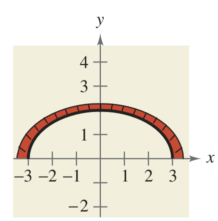
63. ARCHITECTURE A semielliptical arch over a tunnel for a one-way road through a mountain has a major axis of 50 feet and a height at the center of 10 feet.

(a) Draw a rectangular coordinate system on a sketch of the tunnel with the center of the road entering the tunnel at the origin. Identify the coordinates of the known points.

(b) Find an equation of the semielliptical arch.

(c) You are driving a moving truck that has a width of 8 feet and a height of 9 feet. Will the moving truck clear the opening of the arch?

64. ARCHITECTURE A fireplace arch is to be constructed in the shape of a semiellipse. The opening is to have a height of 2 feet at the center and a width of 6 feet along the base (see figure). The contractor draws the outline of the ellipse using tacks as described at the beginning of this section. Determine the required positions of the tacks and the length of the string.



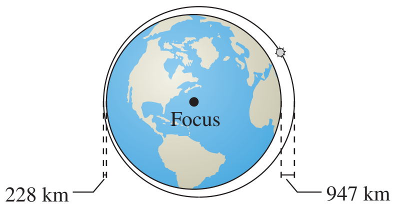
65. COMET ORBIT Halley's comet has an elliptical orbit, with the sun at one focus. The eccentricity of the orbit is approximately 0.967. The length of the major axis of the orbit is approximately 35.88 astronomical units. (An astronomical unit is about 93 million miles.)

(a) Find an equation of the orbit. Place the center of the orbit at the origin, and place the major axis on the *x*-axis.

(b) Use a graphing utility to graph the equation of the orbit.

(c) Find the greatest (aphelion) and smallest (perihelion) distances from the sun's center to the comet's center.

66. SATELLITE ORBIT The first artificial satellite to orbit Earth was Sputnik I (launched by the former Soviet Union in 1957). Its highest point above Earth's surface was 947 kilometers, and its lowest point was 228 kilometers (see figure). The center of Earth was at one focus of the elliptical orbit, and the radius of Earth is 6378 kilometers. Find the eccentricity of the orbit.



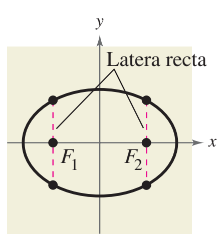
67. MOTION OF A PENDULUM The relation between the velocity y (in radians per second) of a pendulum and its angular displacement θ from the vertical can be modeled by a semiellipse. A 12-centimeter pendulum crests $(y = 0)$ when the angular displacement is -0.2 radian and 0.2 radian. When the pendulum is at equilibrium $(\theta = 0)$, the velocity is -1.6 radians per second.

(a) Find an equation that models the motion of the pendulum. Place the center at the origin.

(b) Graph the equation from part (a).

(c) Which half of the ellipse models the motion of the pendulum?

68. GEOMETRY A line segment through a focus of an ellipse with endpoints on the ellipse and perpendicular to the major axis is called a **latus rectum** of the ellipse. Therefore, an ellipse has two latera recta. Knowing the length of the latera recta is helpful in sketching an ellipse because it yields other points on the curve (see figure). Show that the length of each latus rectum is $2b^2/a$.



In Exercises 69–72, sketch the graph of the ellipse, using latera recta (see Exercise 68).

69. $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$

70. $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$

71. $5x^2 + 3y^2 = 15$

72. $9x^2 + 4y^2 = 36$

EXPLORATION

TRUE OR FALSE? In Exercises 73 and 74, determine whether the statement is true or false. Justify your answer.

73. The graph of $x^2 + 4y^4 - 4 = 0$ is an ellipse.

74. It is easier to distinguish the graph of an ellipse from the graph of a circle if the eccentricity of the ellipse is large (close to 1).

75. Consider the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a + b = 20.$$

(a) The area of the ellipse is given by $A = \pi ab$. Write the area of the ellipse as a function of a.

(b) Find the equation of an ellipse with an area of 264 square centimeters.

(c) Complete the table using your equation from part (a), and make a conjecture about the shape of the ellipse with maximum area.

a	8	9	10	11	12	13
A						

(d) Use a graphing utility to graph the area function and use the graph to support your conjecture in part (c).

76. THINK ABOUT IT At the beginning of this section it was noted that an ellipse can be drawn using two thumbtacks, a string of fixed length (greater than the distance between the two tacks), and a pencil. If the ends of the string are fastened at the tacks and the string is drawn taut with a pencil, the path traced by the pencil is an ellipse.

(a) What is the length of the string in terms of a?

(b) Explain why the path is an ellipse.

77. THINK ABOUT IT Find the equation of an ellipse such that for any point on the ellipse, the sum of the distances from the point $(2, 2)$ and $(10, 2)$ is 36.

78. CAPSTONE Describe the relationship between circles and ellipses. How are they similar? How do they differ?

79. PROOF Show that $a^2 = b^2 + c^2$ for the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where $a > 0$, $b > 0$, and the distance from the center of the ellipse $(0, 0)$ to a focus is c.