10.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. A ________ is the intersection of a plane and a double-napped cone.

2. When a plane passes through the vertex of a double-napped cone, the intersection is a ________ ________.

3. A collection of points satisfying a geometric property can also be referred to as a ________ of points.

4. A ________ is defined as the set of all points (x, y) in a plane that are equidistant from a fixed line, called the ________, and a fixed point, called the ________, not on the line.

5. The line that passes through the focus and the vertex of a parabola is called the ________ of the parabola.

6. The ________ of a parabola is the midpoint between the focus and the directrix.

7. A line segment that passes through the focus of a parabola and has endpoints on the parabola is called a ________ ________ .

8. A line is ________ to a parabola at a point on the parabola if the line intersects, but does not cross, the parabola at the point.

SKILLS AND APPLICATIONS

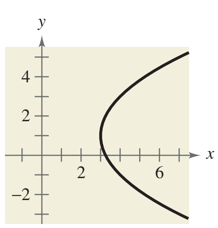
In Exercises 9–12, describe in words how a plane could intersect with the double-napped cone shown to form the conic section.

9. Circle **10.** Ellipse

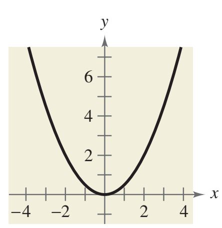
11. Parabola **12.** Hyperbola

In Exercises 13–18, match the equation with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

(a)



(b)



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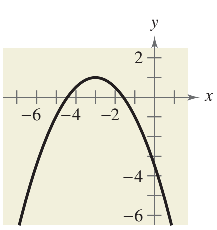

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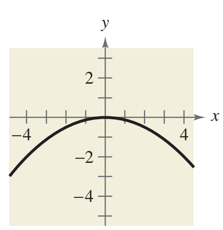

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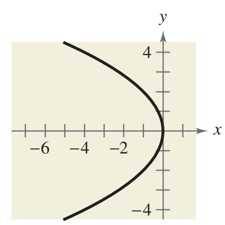
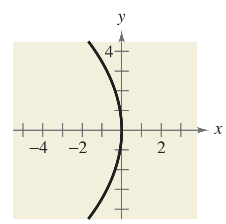


(f)



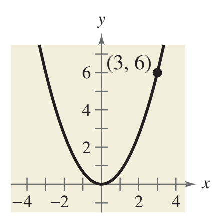
13. $y^2 = -4x$ **14.** $x^2 = 2y$

15. $x^2 = -8y$ **16.** $y^2 = -12x$

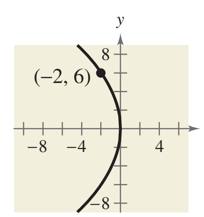
17. $(y - 1)^2 = 4(x - 3)$ **18.** $(x + 3)^2 = -2(y - 1)$

In Exercises 19–32, find the standard form of the equation of the parabola with the given characteristic(s) and vertex at the origin.

19.



20.



21. Focus: $\left(0, \frac{1}{2}\right)$ **22.** Focus: $\left(-\frac{3}{2}, 0\right)$

23. Focus: $(-2, 0)$ **24.** Focus: $(0, -2)$

25. Directrix: $y = 1$ **26.** Directrix: $y = -2$

27. Directrix: $x = -1$ **28.** Directrix: $x = 3$

29. Vertical axis and passes through the point $(4, 6)$

30. Vertical axis and passes through the point $(-3, -3)$

31. Horizontal axis and passes through the point $(-2, 5)$

32. Horizontal axis and passes through the point $(3, -2)$

In Exercises 33–46, find the vertex, focus, and directrix of the parabola, and sketch its graph.

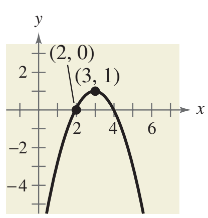
33. $y = \frac{1}{2}x^2$
34. $y = -2x^2$
35. $y^2 = -6x$
36. $y^2 = 3x$
37. $x^2 + 6y = 0$
38. $x + y^2 = 0$
39. $(x - 1)^2 + 8(y + 2) = 0$
40. $(x + 5) + (y - 1)^2 = 0$
41. $(x + 3)^2 = 4\left(y - \frac{3}{2}\right)$
42. $\left(x + \frac{1}{2}\right)^2 = 4(y - 1)$
43. $y = \frac{1}{4}(x^2 - 2x + 5)$
44. $x = \frac{1}{4}(y^2 + 2y + 33)$
45. $y^2 + 6y + 8x + 25 = 0$
46. $y^2 - 4y - 4x = 0$

In Exercises 47–50, find the vertex, focus, and directrix of the parabola. Use a graphing utility to graph the parabola.

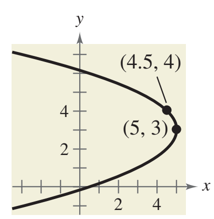
47. $x^2 + 4x + 6y - 2 = 0$
48. $x^2 - 2x + 8y + 9 = 0$
49. $y^2 + x + y = 0$
50. $y^2 - 4x - 4 = 0$

In Exercises 51–60, find the standard form of the equation of the parabola with the given characteristics.

51.



52.



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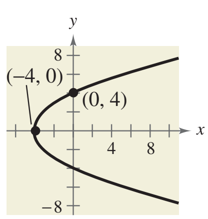
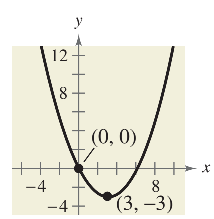


54.



55. Vertex: $(4, 3)$; focus: $(6, 3)$
56. Vertex: $(-1, 2)$; focus: $(-1, 0)$
57. Vertex: $(0, 2)$; directrix: $y = 4$
58. Vertex: $(1, 2)$; directrix: $y = -1$
59. Focus: $(2, 2)$; directrix: $x = -2$
60. Focus: $(0, 0)$; directrix: $y = 8$

In Exercises 61 and 62, change the equation of the parabola so that its graph matches the description.

61. $(y - 3)^2 = 6(x + 1)$; upper half of parabola
62. $(y + 1)^2 = 2(x - 4)$; lower half of parabola

In Exercises 63 and 64, the equations of a parabola and a tangent line to the parabola are given. Use a graphing utility to graph both equations in the same viewing window. Determine the coordinates of the point of tangency.

	Parabola	*Tangent Line*
63.	$y^2 - 8x = 0$	$x - y + 2 = 0$
64.	$x^2 + 12y = 0$	$x + y - 3 = 0$

In Exercises 65–68, find an equation of the tangent line to the parabola at the given point, and find the x-intercept of the line.

65. $x^2 = 2y$, $(4, 8)$
66. $x^2 = 2y$, $\left(-3, \frac{9}{2}\right)$
67. $y = -2x^2$, $(-1, -2)$
68. $y = -2x^2$, $(2, -8)$

69. REVENUE The revenue R (in dollars) generated by the sale of x units of a patio furniture set is given by

$$(x - 106)^2 = -\frac{4}{5}(R - 14{,}045).$$

Use a graphing utility to graph the function and approximate the number of sales that will maximize revenue.

70. REVENUE The revenue R (in dollars) generated by the sale of x units of a digital camera is given by

$$(x - 135)^2 = -\frac{5}{7}(R - 25{,}515).$$

Use a graphing utility to graph the function and approximate the number of sales that will maximize revenue.

71. SUSPENSION BRIDGE Each cable of the Golden Gate Bridge is suspended (in the shape of a parabola) between two towers that are 1280 meters apart. The top of each tower is 152 meters above the roadway. The cables touch the roadway midway between the towers.

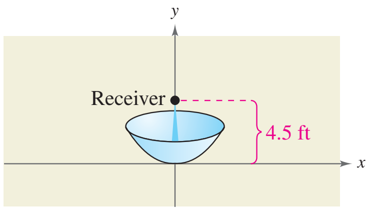
(a) Draw a sketch of the bridge. Locate the origin of a rectangular coordinate system at the center of the roadway. Label the coordinates of the known points.

(b) Write an equation that models the cables.

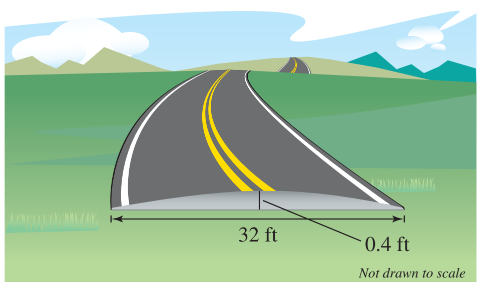
(c) Complete the table by finding the height y of the suspension cables over the roadway at a distance of x meters from the center of the bridge.

Distance, x	Height, y
0	
100	
250	
400	
500	

72. SATELLITE DISH The receiver in a parabolic satellite dish is 4.5 feet from the vertex and is located at the focus (see figure). Write an equation for a cross section of the reflector. (Assume that the dish is directed upward and the vertex is at the origin.)



73. ROAD DESIGN Roads are often designed with parabolic surfaces to allow rain to drain off. A particular road that is 32 feet wide is 0.4 foot higher in the center than it is on the sides (see figure).

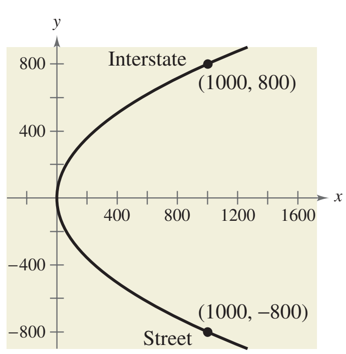


Cross section of road surface

(a) Find an equation of the parabola that models the road surface. (Assume that the origin is at the center of the road.)

(b) How far from the center of the road is the road surface 0.1 foot lower than in the middle?

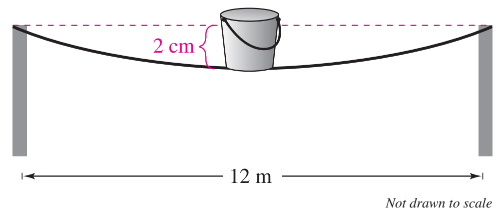
74. HIGHWAY DESIGN Highway engineers design a parabolic curve for an entrance ramp from a straight street to an interstate highway (see figure). Find an equation of the parabola.



75. BEAM DEFLECTION A simply supported beam is 12 meters long and has a load at the center (see figure). The deflection of the beam at its center is 2 centimeters. Assume that the shape of the deflected beam is parabolic.

(a) Write an equation of the parabola. (Assume that the origin is at the center of the deflected beam.)

(b) How far from the center of the beam is the deflection equal to 1 centimeter?



76. BEAM DEFLECTION Repeat Exercise 75 if the length of the beam is 16 meters and the deflection of the beam at the center is 3 centimeters.

77. FLUID FLOW Water is flowing from a horizontal pipe 48 feet above the ground. The falling stream of water has the shape of a parabola whose vertex $(0, 48)$ is at the end of the pipe (see figure). The stream of water strikes the ground at the point $\left(10\sqrt{3}, 0\right)$. Find the equation of the path taken by the water.

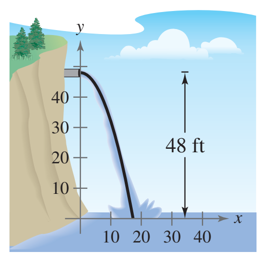


FIGURE FOR 77

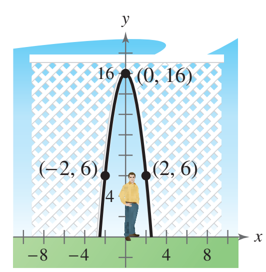


FIGURE FOR 78

78. LATTICE ARCH A parabolic lattice arch is 16 feet high at the vertex. At a height of 6 feet, the width of the lattice arch is 4 feet (see figure). How wide is the lattice arch at ground level?

79. SATELLITE ORBIT A satellite in a 100-mile-high circular orbit around Earth has a velocity of approximately 17,500 miles per hour. If this velocity is multiplied by $\sqrt{2}$, the satellite will have the minimum velocity necessary to escape Earth's gravity and it will follow a parabolic path with the center of Earth as the focus (see figure on the next page).

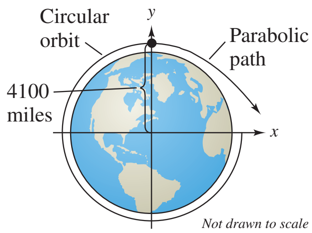


FIGURE FOR 79

(a) Find the escape velocity of the satellite.

(b) Find an equation of the parabolic path of the satellite (assume that the radius of Earth is 4000 miles).

80. PATH OF A SOFTBALL The path of a softball is modeled by $-12.5(y - 7.125) = (x - 6.25)^2$, where the coordinates x and y are measured in feet, with $x = 0$ corresponding to the position from which the ball was thrown.

(a) Use a graphing utility to graph the trajectory of the softball.

(b) Use the *trace* feature of the graphing utility to approximate the highest point and the range of the trajectory.

PROJECTILE MOTION In Exercises 81 and 82, consider the path of a projectile projected horizontally with a velocity of v feet per second at a height of s feet, where the model for the path is

$$x^2 = -\frac{v^2}{16}(y - s).$$

In this model (in which air resistance is disregarded), y is the height (in feet) of the projectile and x is the horizontal distance (in feet) the projectile travels.

81. A ball is thrown from the top of a 100-foot tower with a velocity of 28 feet per second.

(a) Find the equation of the parabolic path.

(b) How far does the ball travel horizontally before striking the ground?

82. A cargo plane is flying at an altitude of 30,000 feet and a speed of 540 miles per hour. A supply crate is dropped from the plane. How many *feet* will the crate travel horizontally before it hits the ground?

EXPLORATION

TRUE OR FALSE? In Exercises 83 and 84, determine whether the statement is true or false. Justify your answer.

83. It is possible for a parabola to intersect its directrix.

84. If the vertex and focus of a parabola are on a horizontal line, then the directrix of the parabola is vertical.

85. Let (x_1, y_1) be the coordinates of a point on the parabola $x^2 = 4py$. The equation of the line tangent to the parabola at the point is

$$y - y_1 = \frac{x_1}{2p}(x - x_1).$$

What is the slope of the tangent line?

86. CAPSTONE Explain what each of the following equations represents, and how equations (a) and (b) are equivalent.

(a) $y = a(x - h)^2 + k, \quad a \neq 0$

(b) $(x - h)^2 = 4p(y - k), \quad p \neq 0$

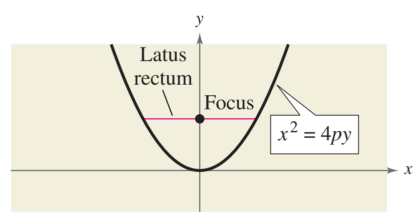
(c) $(y - k)^2 = 4p(x - h), \quad p \neq 0$

87. GRAPHICAL REASONING Consider the parabola $x^2 = 4py$.

(a) Use a graphing utility to graph the parabola for $p = 1$, $p = 2$, $p = 3$, and $p = 4$. Describe the effect on the graph when p increases.

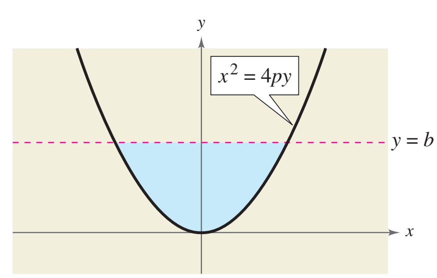
(b) Locate the focus for each parabola in part (a).

(c) For each parabola in part (a), find the length of the latus rectum (see figure). How can the length of the latus rectum be determined directly from the standard form of the equation of the parabola?



(d) Explain how the result of part (c) can be used as a sketching aid when graphing parabolas.

88. GEOMETRY The area of the shaded region in the figure is $A = \frac{8}{3}p^{1/2}b^{3/2}$.



(a) Find the area when $p = 2$ and $b = 4$.

(b) Give a geometric explanation of why the area approaches 0 as p approaches 0.