12.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. If $f(x)$ becomes arbitrarily close to a unique number L as x approaches c from either side, the _______ of $f(x)$ as x approaches c is L.
2. An alternative notation for $\lim_{x\to c} f(x) = L$ is $f(x) \to L$ as $x \to c$, which is read as "$f(x)$ _______ L as x _______ c."
3. The limit of $f(x)$ as $x \to c$ does not exist if $f(x)$ _______ between two fixed values.
4. To evaluate the limit of a polynomial function, use _______ _______.

SKILLS AND APPLICATIONS

5. **GEOMETRY** You create an open box from a square piece of material 24 centimeters on a side. You cut equal squares from the corners and turn up the sides.
   (a) Draw and label a diagram that represents the box.
   (b) Verify that the volume V of the box is given by
   $V = 4x(12 - x)^2$.
   (c) The box has a maximum volume when $x = 4$. Use a graphing utility to complete the table and observe the behavior of the function as x approaches 4. Use the table to find $\lim_{x\to 4} V$.

x	3	3.5	3.9	4	4.1	4.5	5
V							

   (d) Use a graphing utility to graph the volume function. Verify that the volume is maximum when $x = 4$.

6. **GEOMETRY** You are given wire and are asked to form a right triangle with a hypotenuse of $\sqrt{18}$ inches whose area is as large as possible.
   (a) Draw and label a diagram that shows the base x and height y of the triangle.
   (b) Verify that the area A of the triangle is given by
   $A = \frac{1}{2}x\sqrt{18 - x^2}$.
   (c) The triangle has a maximum area when $x = 3$ inches. Use a graphing utility to complete the table and observe the behavior of the function as x approaches 3. Use the table to find $\lim_{x\to 3} A$.

x	2	2.5	2.9	3	3.1	3.5	4
A							

   (d) Use a graphing utility to graph the area function. Verify that the area is maximum when $x = 3$ inches.

In Exercises 7–12, complete the table and use the result to estimate the limit numerically. Determine whether or not the limit can be reached.

7. $\lim_{x\to 2} (5x + 4)$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				?			

8. $\lim_{x\to 1} (2x^2 + x - 4)$

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$				?			

9. $\lim_{x\to 3} \dfrac{x - 3}{x^2 - 9}$

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$				?			

10. $\lim_{x\to -1} \dfrac{x + 1}{x^2 - x - 2}$

x	-1.1	-1.01	-1.001	-1	-0.999
$f(x)$				?	

x	-0.99	-0.9
$f(x)$		

11. $\lim_{x\to 0} \dfrac{\sin 2x}{x}$

x	-0.1	-0.01	-0.001	0	0.001
$f(x)$				?	

x	0.01	0.1
$f(x)$		

12. $\lim_{x\to 0} \frac{\tan x}{2x}$

x	-0.1	-0.01	-0.001	0	0.001
$f(x)$				?	

x	0.01	0.1
$f(x)$		

In Exercises 13–26, create a table of values for the function and use the result to estimate the limit numerically. Use a graphing utility to graph the corresponding function to confirm your result graphically.

13. $\lim_{x\to 1} \frac{x-1}{x^2+2x-3}$

14. $\lim_{x\to -2} \frac{x+2}{x^2+5x+6}$

15. $\lim_{x\to 0} \frac{\sqrt{x+5}-\sqrt{5}}{x}$

16. $\lim_{x\to -3} \frac{\sqrt{1-x}-2}{x+3}$

17. $\lim_{x\to -4} \frac{\frac{x}{x+2}-2}{x+4}$

18. $\lim_{x\to 2} \frac{\frac{1}{x+2}-\frac{1}{4}}{x-2}$

19. $\lim_{x\to 0} \frac{\sin x}{x}$

20. $\lim_{x\to 0} \frac{\cos x - 1}{x}$

21. $\lim_{x\to 0} \frac{\sin^2 x}{x}$

22. $\lim_{x\to 0} \frac{2x}{\tan 4x}$

23. $\lim_{x\to 0} \frac{e^{2x}-1}{2x}$

24. $\lim_{x\to 0} \frac{1-e^{-4x}}{x}$

25. $\lim_{x\to 1} \frac{\ln(2x-1)}{x-1}$

26. $\lim_{x\to 1} \frac{\ln(x^2)}{x-1}$

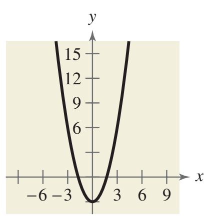
In Exercises 27 and 28, graph the function and find the limit (if it exists) as x approaches 2.

27. $f(x) = \begin{cases} 2x+1, & x < 2 \\ x+3, & x \geq 2 \end{cases}$

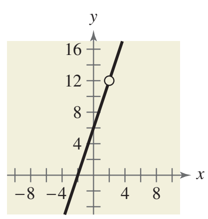
28. $f(x) = \begin{cases} -2x, & x \leq 2 \\ x^2-4x+1, & x > 2 \end{cases}$

In Exercises 29–36, use the graph to find the limit (if it exists). If the limit does not exist, explain why.

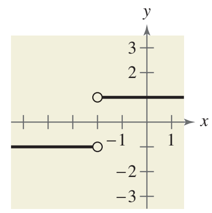
29. $\lim_{x\to -4} (x^2-3)$



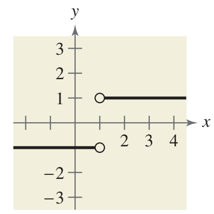
30. $\lim_{x\to 2} \frac{3x^2-12}{x-2}$



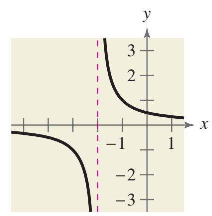
31. $\lim_{x\to -2} \frac{|x+2|}{x+2}$



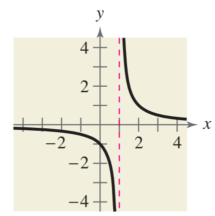
32. $\lim_{x\to 1} \frac{|x-1|}{x-1}$



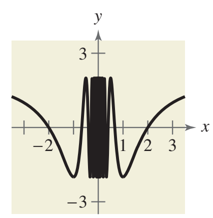
33. $\lim_{x\to -2} \frac{x-2}{x^2-4}$



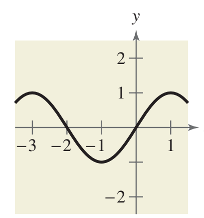
34. $\lim_{x\to 1} \frac{1}{x-1}$



35. $\lim_{x\to 0} 2\cos\frac{\pi}{x}$



36. $\lim_{x\to -1} \sin\frac{\pi x}{2}$



In Exercises 37–44, use a graphing utility to graph the function and use the graph to determine whether the limit exists. If the limit does not exist, explain why.

37. $f(x) = \frac{5}{2+e^{1/x}}, \quad \lim_{x\to 0} f(x)$

38. $f(x) = \ln(7-x), \quad \lim_{x\to -1} f(x)$

39. $f(x) = \cos\frac{1}{x}, \quad \lim_{x\to 0} f(x)$

40. $f(x) = \sin \pi x, \quad \lim_{x\to 1} f(x)$

41. $f(x) = \frac{\sqrt{x+3}-1}{x-4}, \quad \lim_{x\to 4} f(x)$

42. $f(x) = \frac{\sqrt{x+5}-4}{x-2}, \quad \lim_{x\to 2} f(x)$

43. $f(x) = \frac{x-1}{x^2-4x+3}, \quad \lim_{x\to 1} f(x)$

44. $f(x) = \frac{7}{x-3}, \quad \lim_{x\to 3} f(x)$

In Exercises 45 and 46, use the given information to evaluate each limit.

45. $\lim_{x\to c} f(x) = 3, \quad \lim_{x\to c} g(x) = 6$

(a) $\lim_{x\to c} [-2g(x)]$ (b) $\lim_{x\to c} [f(x) + g(x)]$

(c) $\lim_{x\to c} \dfrac{f(x)}{g(x)}$ (d) $\lim_{x\to c} \sqrt{f(x)}$

46. $\lim_{x\to c} f(x) = 5, \quad \lim_{x\to c} g(x) = -2$

(a) $\lim_{x\to c} [f(x) + g(x)]^2$ (b) $\lim_{x\to c} [6f(x)g(x)]$

(c) $\lim_{x\to c} \dfrac{5g(x)}{4f(x)}$ (d) $\lim_{x\to c} \dfrac{1}{\sqrt{f(x)}}$

In Exercises 47 and 48, find (a) $\lim_{x\to 2} f(x)$, (b) $\lim_{x\to 2} g(x)$, (c) $\lim_{x\to 2} [f(x)g(x)]$, and (d) $\lim_{x\to 2} [g(x) - f(x)]$.

47. $f(x) = x^3, \quad g(x) = \dfrac{\sqrt{x^2 + 5}}{2x^2}$

48. $f(x) = \dfrac{x}{3 - x}, \quad g(x) = \sin \pi x$

In Exercises 49–68, find the limit by direct substitution.

49. $\lim_{x\to 5} (10 - x^2)$

50. $\lim_{x\to -2} \left(\frac{1}{2}x^3 - 5x\right)$

51. $\lim_{x\to -3} (2x^2 + 4x + 1)$

52. $\lim_{x\to -2} (x^3 - 6x + 5)$

53. $\lim_{x\to 3} \left(-\dfrac{9}{x}\right)$

54. $\lim_{x\to -5} \dfrac{6}{x + 2}$

55. $\lim_{x\to -3} \dfrac{3x}{x^2 + 1}$

56. $\lim_{x\to 4} \dfrac{x - 1}{x^2 + 2x + 3}$

57. $\lim_{x\to -2} \dfrac{5x + 3}{2x - 9}$

58. $\lim_{x\to 3} \dfrac{x^2 + 1}{x}$

59. $\lim_{x\to -1} \sqrt{x + 2}$

60. $\lim_{x\to 3} \sqrt[3]{x^2 - 1}$

61. $\lim_{x\to 7} \dfrac{5x}{\sqrt{x + 2}}$

62. $\lim_{x\to 8} \dfrac{\sqrt{x + 1}}{x - 4}$

63. $\lim_{x\to 3} e^x$

64. $\lim_{x\to e} \ln x$

65. $\lim_{x\to \pi} \sin 2x$

66. $\lim_{x\to \pi} \tan x$

67. $\lim_{x\to 1/2} \arcsin x$

68. $\lim_{x\to 1} \arccos \dfrac{x}{2}$

EXPLORATION

TRUE OR FALSE? **In Exercises 69 and 70, determine whether the statement is true or false. Justify your answer.**

69. The limit of a function as x approaches c does not exist if the function approaches -3 from the left of c and 3 from the right of c.

70. The limit of the product of two functions is equal to the product of the limits of the two functions.

71. THINK ABOUT IT From Exercises 7–12, select a limit that can be reached and one that cannot be reached.

(a) Use a graphing utility to graph the corresponding functions using a standard viewing window. Do the graphs reveal whether or not the limit can be reached? Explain.

(b) Use a graphing utility to graph the corresponding functions using a *decimal* setting. Do the graphs reveal whether or not the limit can be reached? Explain.

72. THINK ABOUT IT Use the results of Exercise 71 to draw a conclusion as to whether or not you can use the graph generated by a graphing utility to determine reliably if a limit can be reached.

73. THINK ABOUT IT

(a) If $f(2) = 4$, can you conclude anything about $\lim_{x\to 2} f(x)$? Explain your reasoning.

(b) If $\lim_{x\to 2} f(x) = 4$, can you conclude anything about $f(2)$? Explain your reasoning.

74. WRITING Write a brief description of the meaning of the notation $\lim_{x\to 5} f(x) = 12$.

75. THINK ABOUT IT Use a graphing utility to graph the tangent function. What are $\lim_{x\to 0} \tan x$ and $\lim_{x\to \pi/4} \tan x$? What can you say about the existence of the limit $\lim_{x\to \pi/2} \tan x$?

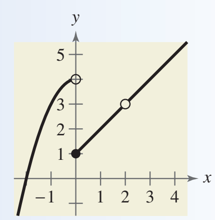
76. CAPSTONE Use the graph of the function f to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.

(a) $f(0)$

(b) $\lim_{x\to 0} f(x)$

(c) $f(2)$

(d) $\lim_{x\to 2} f(x)$



77. WRITING Use a graphing utility to graph the function given by $f(x) = \dfrac{x^2 - 3x - 10}{x - 5}$. Use the *trace* feature to approximate $\lim_{x\to 4} f(x)$. What do you think $\lim_{x\to 5} f(x)$ equals? Is f defined at $x = 5$? Does this affect the existence of the limit as x approaches 5?