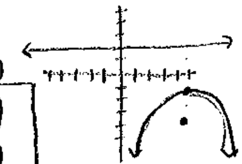
Honors Pre-Calculus
Unit 6 Study Guide
Conics

1. Find the vertex, focus, and directrix of the parabola: $x^2 - 10x + 12y + 37 = 0$



$x^2 - 10x + 25 = -12y - 37 + 25$

$(x-5)^2 = -12y - 12$

$(x-5)^2 = -12(y+1)$

$-12 = 4p \quad p = -3$

$\boxed{\text{vertex: } (5,-1) \quad \text{focus: } (5,-4) \quad \text{directrix: } y = 2}$

2. Find the vertex, focus, and directrix of the parabola: $4y^2 - 4y - 4x + 5 = 0$

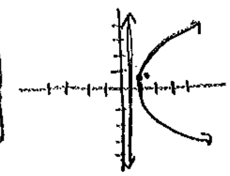
$\frac{4y^2}{4} - \frac{4y}{4} = \frac{4x}{4} - \frac{5}{4}$

$\left(y^2 - y + \frac{1}{4}\right) = x - \frac{5}{4} + \frac{1}{4}$

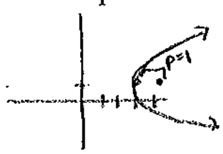
$\left(y - \frac{1}{2}\right)^2 = x - 1$

$4p = 1$
$p = 1/4$

$\boxed{\text{vertex: } (1, 1/2) \quad \text{focus: } (5/4, 1/2) \quad \text{directrix: } x = 3/4}$



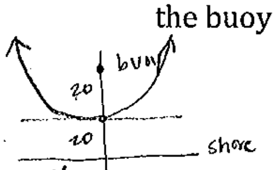
3. Write the equation of a parabola in standard form with a vertex at (3, 1) and a focus at (4, 1).

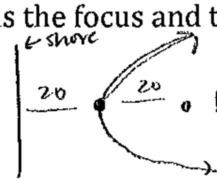


$\boxed{(y-1)^2 = 4(x-3)}$ $\quad (y-k)^2 = 4p(x-h)$

$p = 1$

4. The course for a sailboat race includes a turn-around point marked by a stationary buoy. The sailboats must pass between the buoy and the straight shoreline. The boats follow a parabolic path past the buoy, which is 40 yards from the shoreline. Find an equation to represent the parabolic path so that the boats remain equidistant from the buoy and the straight shoreline, using the buoy as the focus and the shoreline as the directrix.





$x^2 = 4py$ or $y^2 = 4px$

$x^2 = 4(20)y$

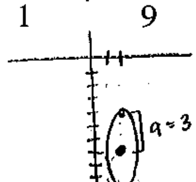
$\boxed{x^2 = 80y}$ or $\boxed{y^2 = 80x}$

* a is always bigger

$a = \sqrt{9} = 3$
$b = \sqrt{1} = 1$

5. Find the center, vertices, foci, and eccentricity of the ellipse:

$$\frac{(x-2)^2}{1} + \frac{(y+7)^2}{9} = 1$$



center: $(2,-7)$
vertices: $(2,-4)$ $(2,-10)$
foci: $(2, -7+\sqrt{8})$ $(2, -7-\sqrt{8})$
eccentricity: $\frac{\sqrt{8}}{3}$

$c^2 = a^2 - b^2$
$c^2 = 9 - 1 \quad c^2 = 8 \quad c = \sqrt{8}$

focus: c away from center
eccentricity: c/a

vertices are a away from center

6. Find the vertices of the ellipse: $2x^2 + y^2 + 20x + 48 = 0$

center: $(-5, 0)$
$a = \sqrt{2}$

$\boxed{\text{vertices: } (-5, \sqrt{2}) \quad (-5, -\sqrt{2})}$

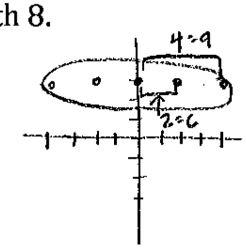
$(2x^2 + 20x \quad) + y^2 = -48$

$2(x^2 + 10x + 25) + y^2 = -48 + 50$

$2(x+5)^2 + y^2 = 2$

$\frac{(x+5)^2}{1} + \frac{y^2}{2} = 1$

7. Find an equation of the ellipse with foci (-2, 3) and (2, 3) and major axis of length 8.



center: $(0,3)$
$a = 4$
$c = 2$
$b = \sqrt{12}$

$c^2 = a^2 - b^2$
$4 = 16 - b^2$
$b^2 = 12$

$$\boxed{\frac{(x-0)^2}{16} + \frac{(y-3)^2}{12} = 1}$$

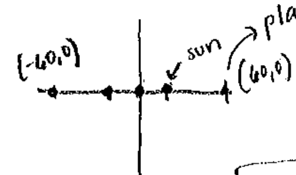
7

8. For a science project, you plan to demonstrate the elliptical orbit of a planet with the sun at one of the foci. If the center of the ellipse is (0, 0), and the length of the major axis is 120 cm, what is the smallest distance from the planet to the sun, if the sun is located at the point (1, 0)?

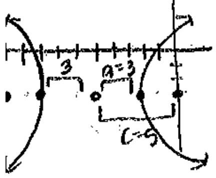
$c = ?$

$a = 60$ cm

$\boxed{59 \text{ cm}}$



opens left/right because y is 2nd



9. Find the center, vertices, foci, and asymptotes of the hyperbola: $16x^2 - 9y^2 + 160x - 54y + 175 = 0$.

$(16x^2 + 160x \quad) + (-9y^2 - 54y \quad) = -175$

$16(x^2 + 10x + 25) - 9(y^2 + 6y + 9) = -175 + 400 - 81$

$16(x+5)^2 - 9(y+3)^2 = 144$

$\frac{(x+5)^2}{9} - \frac{(y+3)^2}{16} = 1$

$c^2 = a^2 + b^2 \quad c^2 = 3^2 + 4^2 \quad c^2 = 25 \quad c = 5 \quad a = 3 \quad b = 4$

center: $(-5, -3)$
vertices: $(-2, -3)$, $(-8, -3)$
foci: $(0, -3)$, $(-10, -3)$
asymptotes: $y = -3 \pm \frac{4}{3}(x+5)$

opens up/down because y is 1st

$h = 0$
$k = -2$

10. Find the standard form of the equation of the hyperbola with center (0, -2), one vertex (0, -4), and one focus (0, 2).

$a = 2 \quad c = 4$

$4^2 = 2^2 + b^2$
$16 + 4 = b^2$
$b = \sqrt{12}$

$\boxed{\frac{(y+2)^2}{4} - \frac{(x-0)^2}{12} = 1}$

11. Find the center and radius of the circle with equation: $3x^2 + 3y^2 - 6x + 18y - 7 = 0$

$(3x^2 - 6x \quad) + (3y^2 + 18y \quad) = 7$

$3(x^2 - 2x + 1) + 3(y^2 + 6y + 9) = 7 + 3 + 27$

$3(x-1)^2 + 3(y+3)^2 = 37$

$(x-1)^2 + (y+3)^2 = \frac{37}{3}$

center: $(1, -3)$
radius: $\sqrt{37/3}$

12. Use elimination to solve the system of equations:

$\cancel{-x^2} + y^2 - 6y + 4 = 0$ (+4)

$+ \quad \cancel{x^2} + y^2 - 4x - 6y + 12 = 0$

$2y^2 - 12y + 16 = 0$

$2(y^2 - 6y + 8) = 0$

$2(y-4)(y-2) = 0$

$y - 4 = 0 \quad y - 2 = 0 \quad y = 4, y = 2$

plug in $y = 4$

$x^2 + 4^2 - 4x - 6(4) + 12 = 0$
$x^2 + 16 - 4x - 24 + 12 = 0$
$x^2 - 4x + 4 = 0$
$(x-2)(x-2) = 0$
$x = 2$ $\boxed{(4,2)}$

plug in $y = 2$

$x^2 + 2^2 - 4x - 6(2) + 12 = 0$
$x^2 + 4 - 4x - 12 + 12 = 0$
$x^2 - 4x + 4 = 0$
$(x-2)(x-2) = 0$
$x = 2$ $\boxed{(2,2)}$

For questions #13-15, classify the given graph as a circle, parabola, ellipse, or hyperbola.

13. $2x^2 - 5y^2 + 4x - 6 = 0$ $\boxed{\text{hyperbola}}$

x^2 is positive
y^2 is negative

14. $3x^2 + 3y^2 - 6x + 18y + 10 = 0$ $\boxed{\text{circle}}$

x^2 and y^2 have same coefficient

15. $x^2 + 2x + 4y^2 + 1 = 0$ $\boxed{\text{ellipse}}$

x^2 and y^2 both positive, but different coefficients