CHAPTER 3.1-3.6 REVIEW NAME _____
SHOW ALL WORK!!!

Multiple Choice

1. Find all critical numbers for the function: $f(x) = \frac{x - 1}{x + 3}$.
   (a) 1
   (b) 1, -3
   (c) -3
   (d) 1, -1
   (e) None of these

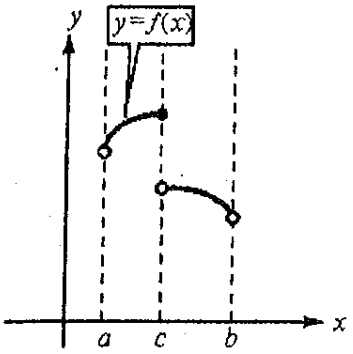
2. Find all extrema in the interval $[0, 2\pi]$ for $y = x + \sin x$.
   (a) $\left(-1, -1 + \frac{3\pi}{2}\right)$
   (b) (π, π)
   (c) $(-1, 0)$
   (d) $\left(\frac{3\pi}{2}, 0\right)$
   (e) None of these

3. Find the absolute maximum and absolute minimum of f on the interval $(-1, 2]$.

$$f(x) = \frac{-x^3 + x^2 + 3x + 1}{x + 1}$$

   (a) Maximum: (1, -2) Minimum: (-1, 2)
   (b) Maximum: (1, -2) Minimum: None
   (c) Maximum: None Minimum: None
   (d) Maximum: None Minimum: (-1, 2)
   (e) None of these

4. Determine from the graph whether f possesses extrema on the interval (a, b).
   (a) Maximum at $x = c$, minimum at $x = b$
   (b) Maximum at $x = c$, no minimum
   (c) No maximum, minimum at $x = b$
   (d) No extrema
   (e) None of these



5. Find all critical numbers for the function $f(x) = (x + 2)^3(x - 1)^4$.
   (a) -2 and 1
   (b) $-2, -\frac{7}{5}, 1$
   (c) -2
   (d) $-2, 1, -\frac{5}{7}$
   (e) None of these

6. Given $f(x) = 8 - \frac{7}{x}$, find all c in the interval $(1, 7)$ such that $f'(c) = \frac{f(7) - f(1)}{7 - 1}$.

(a) $\sqrt{7}$
(b) 4
(c) $\pm\sqrt{7}$
(d) $\frac{7}{8}$
(e) None of these

7. Determine whether the Mean Value Theorem applies to $f(x) = -\frac{1}{x}$ on the interval $\left[-3, -\frac{1}{2}\right]$. If the Mean Value Theorem applies, find all values of c in the interval such that $f'(c) = \frac{f\left[-\frac{1}{2}\right] - f(-3)}{-\frac{1}{2} - (-3)}$. If the Mean Value Theorem does not apply, state why.

(a) Mean Value Theorem applies; $c = -\sqrt{\frac{2}{3}}$.
(b) Mean Value Theorem applies; $c = \pm\sqrt{\frac{2}{3}}$.
(c) The Mean Value Theorem does not apply because f is not continuous at $x = 0$.
(d) The Mean Value Theorem does not apply because $f\left[-\frac{1}{2}\right] \neq f(-3)$.
(e) None of these

8. Find all open intervals on which $f(x) = \frac{x}{x^2 + x - 2}$ is decreasing.
   (a) $(-\infty, \infty)$
   (b) $(-\infty, 0)$
   (c) $(-\infty, -2)$ and $(1, \infty)$
   (d) $(-\infty, -2)$, $(-2, 1)$, and $(1, \infty)$
   (e) None of these

9. Which of the following statements is true of $f(x) = -x^3 - 6x^2 - 9x - 2$?
   (a) f is decreasing on $(-3, -1)$.
   (b) f is increasing on $(-3, -1)$.
   (c) f is increasing on $(-\infty, -3)$.
   (d) f is increasing on $(-2, \infty)$.
   (e) None of these

10. Find the values of x that give relative extrema for the function $f(x) = (x + 1)^2(x - 2)$.
   (a) Relative maximum: $x = -1$; Relative minimum: $x = 1$
   (b) Relative maxima: $x = 1$, $x = 3$; Relative minimum: $x = -1$
   (c) Relative minimum: $x = 2$
   (d) Relative maximum: $x = -1$; Relative minimum: $x = 2$
   (e) None of these

11. Find all points of inflection of the graph of the function $f(x) = x^4 + x^3$.

(a) $(0, 0)$ and $\left[-\frac{1}{2}, -\frac{1}{16}\right]$

(b) $\left[-\frac{1}{2}, -\frac{1}{16}\right]$

(c) $(0, 0)$

(d) $(0, 0)$ and $\left[-\frac{3}{4}, -\frac{27}{256}\right]$

(e) None of these

12. Give the sign of the second derivative of f at the indicated point.

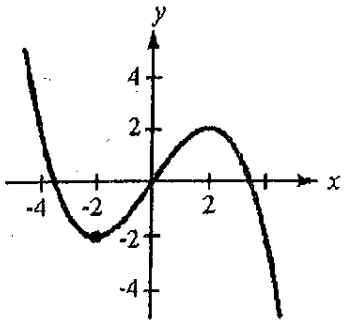
(a) Zero

(b) Negative

(c) Positive

(d) The sign cannot be determined

(e) None of these



13. Which statement is *not* true of the graph of $f(x) = (x + 1)(x - 3)^2$?
    (a) f has a relative maximum at $\left[\frac{1}{3}, \frac{256}{27}\right]$.
    (b) f has a point of inflection at $(3, 0)$.
    (c) f has an intercept at $(3, 0)$.
    (d) f has a relative minimum at $(3, 0)$.
    (e) None of these

14. Let $f(x)$ be a polynomial function such that $f(-2) = 5$, $f'(-2) = 0$, and $f''(-2) = 3$. The point $(-2, 5)$ is a _____ of the graph of f.
    (a) Relative maximum
    (b) Relative minimum
    (c) Intercept
    (d) Point of inflection
    (e) None of these

15. Find the horizontal asymptote: $f(x) = \frac{3x^2 + 2x - 16}{x^2 - 7}$
    (a) $x = \pm\sqrt{7}$
    (b) $y = 3$
    (c) $y = 3x + 7$
    (d) $y = 0$
    (e) None of these

16. Find all horizontal asymptotes for $f(x) = \dfrac{5x}{\sqrt{x^2 + 3}}$.

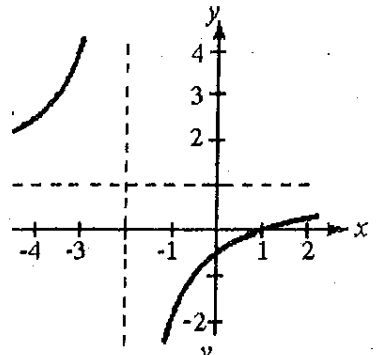
(a) $y = 0$
(b) $y = \pm 5$
(c) $y = 5$
(d) $y = \pm 1$
(e) None of these

17. Find the limit: $\lim\limits_{x \to \infty} \dfrac{a - bx^4}{cx^4 + x^2}$.

(a) 0
(b) ∞
(c) $-\dfrac{b}{c}$
(d) $\dfrac{a}{c}$
(e) None of these

18. Which of the following is the correct sketch of the graph of the function $f(x) = \frac{x - 1}{x + 2}$?

(a)



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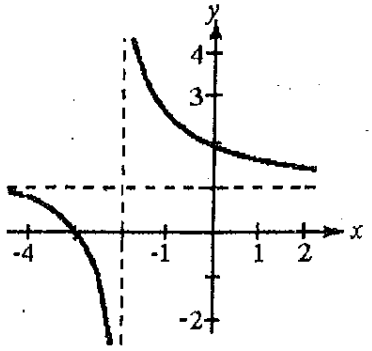

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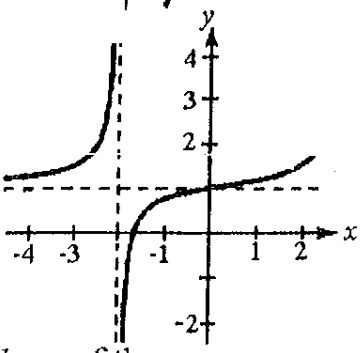
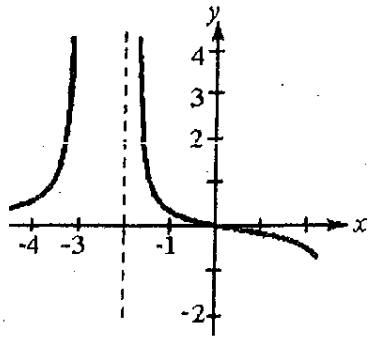


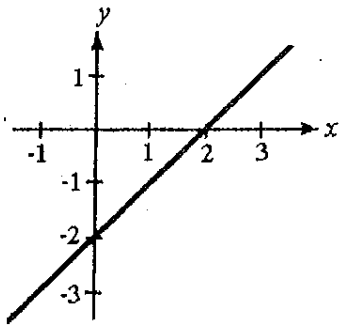
(d)



(e) None of these

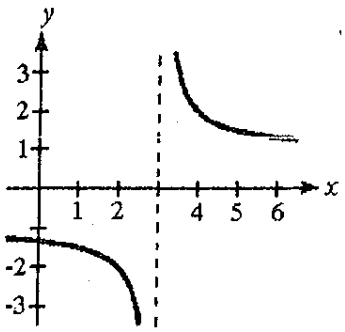
19. Identify the graph of $f(x) = \dfrac{x^2 - 5x + 8}{x - 3}$.

(a)

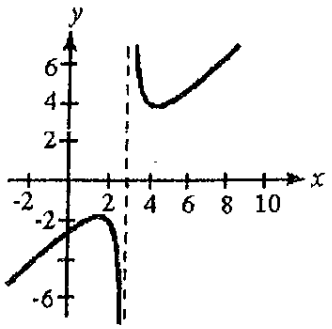


(b) None of these

(c)

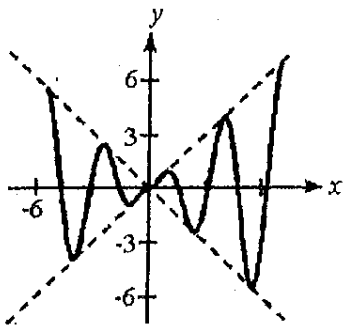


(d)



20. Match the graph with the correct function.

(a) $y = |x| \cos x$
(b) $y = 2^x \sin x$
(c) $y = |x| \sin 2x$
(d) $y = x \sin x$
(e) None of these



21. Consider $f(x) = \frac{x^2}{x^2 + a}$, $a > 0$. Use a graphing utility to determine the effect on the graph of f if a is varied.

(a) Each y value is multiplied by a.
(b) As a increases, the vertical tangent lines move further from the origin.
(c) The graph of the curve is shifted $\sqrt{a}$ units to the left.
(d) As a increases, the curve approaches its asymptote more slowly.
(e) None of these

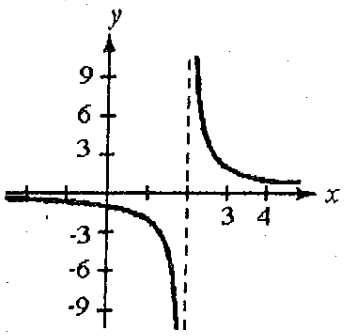
Open-Ended Questions

22. Decide whether Rolle's Theorem can be applied to $f(x) = \frac{x^3 + x}{x}$ on the interval $[-1, 1]$. If Rolle's Theorem can be applied, find all values, c, on the interval such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, state why.

23. Consider $f(x) = \sqrt{x}$. Find all values, c, in the interval $[0, 1]$ such that the slope of the tangent line to the graph of f at c is parallel to the secant line through the points $(0, f(0))$ and $(1, f(1))$.

24. Use a graphing utility to graph $f(x) = 9x^4 + 4x^3 - 36x^2 - 24x$.
    a. Adjust the viewing window and use the zoom and trace features to estimate the x-values of the relative extrema to one decimal place.
    b. Use calculus to find the actual x-values of the relative extrema.

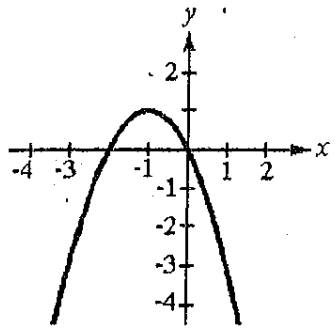
25. Use the graph to identify the open intervals where the function is increasing or decreasing.



26. Find all intervals for which the graph of the function $y = 8x^3 - 2x^4$ is concave downward

27. Find all points of inflection of the graph of the function $f(x) = 2x(x - 4)^3$.

28. The graph of a polynomial function, f, is given. On the same coordinate axes sketch f' and f''.



29. Sketch the graph of a function f such that f' is positive and increasing.

30. Find all horizontal asymptotes: $f(x) = \frac{2}{x - 3} - \frac{x}{x + 2}$.

31. Find the limit: $\lim_{x \to +\infty} \left[\frac{2x}{3x + 1} - \frac{x^2}{x - 3} \right]$.

32. Use the techniques learned in this chapter to sketch the graph of $y = x^3 - 3x + 1$.

33. Use the techniques learned in this chapter to sketch the graph of $f(x) = 3x^{2/3} - x^2$.

34. Use the techniques learned in this chapter to sketch the graph of $f(x) = 2 \sin x + \sin 2x$, $0 \leq x \leq 2\pi$.

35. Use a graphing utility to graph $f(x) = \frac{(-x + 3)(x^2 + x + 2)}{x^2}$. Name any intercepts, relative extrema, points of inflection, or asymptotes.