4.7 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

	Function	*Alternative Notation*	*Domain*	*Range*
1.	$y = \arcsin x$	__________	__________	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
2.	__________	$y = \cos^{-1} x$	$-1 \le x \le 1$	__________
3.	$y = \arctan x$	__________	__________	__________

4. Without restrictions, no trigonometric function has a(n) __________ function.

SKILLS AND APPLICATIONS

In Exercises 5–20, evaluate the expression without using a calculator.

5. $\arcsin \frac{1}{2}$
6. $\arcsin 0$
7. $\arccos \frac{1}{2}$
8. $\arccos 0$
9. $\arctan \frac{\sqrt{3}}{3}$
10. $\arctan(1)$
11. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
12. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
13. $\arctan(-\sqrt{3})$
14. $\arctan \sqrt{3}$
15. $\arccos\left(-\frac{1}{2}\right)$
16. $\arcsin \frac{\sqrt{2}}{2}$
17. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
18. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$
19. $\tan^{-1} 0$
20. $\cos^{-1} 1$

In Exercises 21 and 22, use a graphing utility to graph f, g, and $y = x$ in the same viewing window to verify geometrically that g is the inverse function of f. (Be sure to restrict the domain of f properly.)

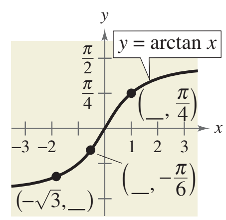
21. $f(x) = \sin x, \quad g(x) = \arcsin x$
22. $f(x) = \tan x, \quad g(x) = \arctan x$

In Exercises 23–40, use a calculator to evaluate the expression. Round your result to two decimal places.

23. $\arccos 0.37$
24. $\arcsin 0.65$
25. $\arcsin(-0.75)$
26. $\arccos(-0.7)$
27. $\arctan(-3)$
28. $\arctan 25$
29. $\sin^{-1} 0.31$
30. $\cos^{-1} 0.26$
31. $\arccos(-0.41)$
32. $\arcsin(-0.125)$
33. $\arctan 0.92$
34. $\arctan 2.8$
35. $\arcsin \frac{7}{8}$
36. $\arccos\left(-\frac{1}{3}\right)$
37. $\tan^{-1} \frac{19}{4}$
38. $\tan^{-1}\left(-\frac{95}{7}\right)$
39. $\tan^{-1}(-\sqrt{372})$
40. $\tan^{-1}(-\sqrt{2165})$

In Exercises 41 and 42, determine the missing coordinates of the points on the graph of the function.

41.



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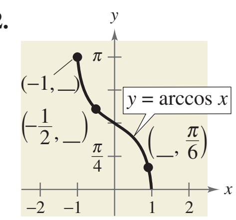

In Exercises 43–48, use an inverse trigonometric function to write θ as a function of x.

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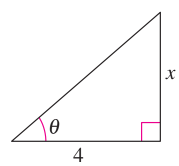

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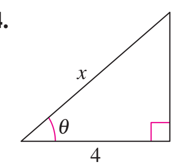

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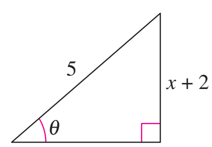

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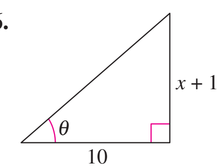

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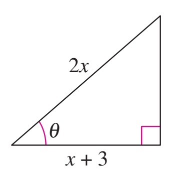

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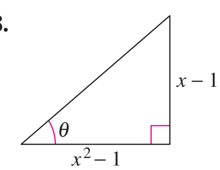


In Exercises 49–54, use the properties of inverse trigonometric functions to evaluate the expression.

49. $\sin(\arcsin 0.3)$
50. $\tan(\arctan 45)$
51. $\cos[\arccos(-0.1)]$
52. $\sin[\arcsin(-0.2)]$
53. $\arcsin(\sin 3\pi)$
54. $\arccos\left(\cos \frac{7\pi}{2}\right)$

In Exercises 55–66, find the exact value of the expression. (*Hint:* Sketch a right triangle.)

55. $\sin\left(\arctan \frac{3}{4}\right)$

56. $\sec\left(\arcsin \frac{4}{5}\right)$

57. $\cos(\tan^{-1} 2)$

58. $\sin\left(\cos^{-1} \frac{\sqrt{5}}{5}\right)$

59. $\cos\left(\arcsin \frac{5}{13}\right)$

60. $\csc\left[\arctan\left(-\frac{5}{12}\right)\right]$

61. $\sec\left[\arctan\left(-\frac{3}{5}\right)\right]$

62. $\tan\left[\arcsin\left(-\frac{3}{4}\right)\right]$

63. $\sin\left[\arccos\left(-\frac{2}{3}\right)\right]$

64. $\cot\left(\arctan \frac{5}{8}\right)$

65. $\csc\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

66. $\sec\left[\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$

In Exercises 67–76, write an algebraic expression that is equivalent to the expression. (*Hint:* Sketch a right triangle, as demonstrated in Example 7.)

67. $\cot(\arctan x)$

68. $\sin(\arctan x)$

69. $\cos(\arcsin 2x)$

70. $\sec(\arctan 3x)$

71. $\sin(\arccos x)$

72. $\sec[\arcsin(x - 1)]$

73. $\tan\left(\arccos \frac{x}{3}\right)$

74. $\cot\left(\arctan \frac{1}{x}\right)$

75. $\csc\left(\arctan \frac{x}{\sqrt{2}}\right)$

76. $\cos\left(\arcsin \frac{x - h}{r}\right)$

In Exercises 77 and 78, use a graphing utility to graph f and g in the same viewing window to verify that the two functions are equal. Explain why they are equal. Identify any asymptotes of the graphs.

77. $f(x) = \sin(\arctan 2x), \quad g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}}$

78. $f(x) = \tan\left(\arccos \frac{x}{2}\right), \quad g(x) = \dfrac{\sqrt{4 - x^2}}{x}$

In Exercises 79–82, fill in the blank.

79. $\arctan \dfrac{9}{x} = \arcsin(\underline{\qquad}), \quad x \neq 0$

80. $\arcsin \dfrac{\sqrt{36 - x^2}}{6} = \arccos(\underline{\qquad}), \quad 0 \leq x \leq 6$

81. $\arccos \dfrac{3}{\sqrt{x^2 - 2x + 10}} = \arcsin(\underline{\qquad})$

82. $\arccos \dfrac{x - 2}{2} = \arctan(\underline{\qquad}), \quad |x - 2| \leq 2$

In Exercises 83 and 84, sketch a graph of the function and compare the graph of g with the graph of $f(x) = \arcsin x$.

83. $g(x) = \arcsin(x - 1)$

84. $g(x) = \arcsin \dfrac{x}{2}$

In Exercises 85–90, sketch a graph of the function.

85. $y = 2 \arccos x$

86. $g(t) = \arccos(t + 2)$

87. $f(x) = \arctan 2x$

88. $f(x) = \dfrac{\pi}{2} + \arctan x$

89. $h(v) = \tan(\arccos v)$

90. $f(x) = \arccos \dfrac{x}{4}$

In Exercises 91–96, use a graphing utility to graph the function.

91. $f(x) = 2 \arccos(2x)$

92. $f(x) = \pi \arcsin(4x)$

93. $f(x) = \arctan(2x - 3)$

94. $f(x) = -3 + \arctan(\pi x)$

95. $f(x) = \pi - \sin^{-1}\left(\dfrac{2}{3}\right)$

96. $f(x) = \dfrac{\pi}{2} + \cos^{-1}\left(\dfrac{1}{\pi}\right)$

In Exercises 97 and 98, write the function in terms of the sine function by using the identity

$$A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \sin\left(\omega t + \arctan \frac{A}{B}\right).$$

Use a graphing utility to graph both forms of the function. What does the graph imply?

97. $f(t) = 3 \cos 2t + 3 \sin 2t$

98. $f(t) = 4 \cos \pi t + 3 \sin \pi t$

In Exercises 99–104, fill in the blank. If not possible, state the reason. (*Note:* The notation $x \rightarrow c^+$ indicates that x approaches c from the right and $x \rightarrow c^-$ indicates that x approaches c from the left.)

99. As $x \rightarrow 1^-$, the value of $\arcsin x \rightarrow \underline{\qquad}$.

100. As $x \rightarrow 1^-$, the value of $\arccos x \rightarrow \underline{\qquad}$.

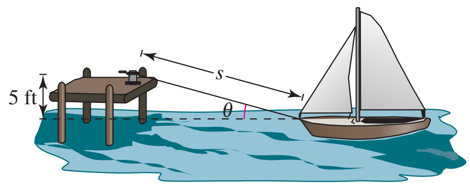
101. As $x \to \infty$, the value of $\arctan x \to$ ____ .

102. As $x \to -1^+$, the value of $\arcsin x \to$ ____ .

103. As $x \to -1^+$, the value of $\arccos x \to$ ____ .

104. As $x \to -\infty$, the value of $\arctan x \to$ ____ .

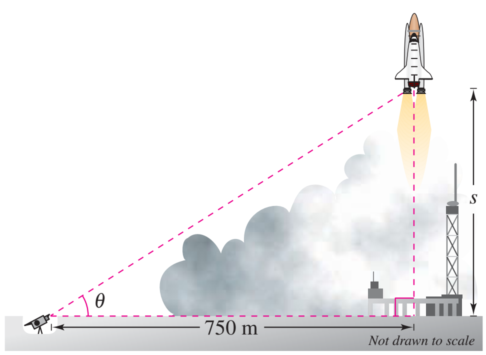
105. DOCKING A BOAT A boat is pulled in by means of a winch located on a dock 5 feet above the deck of the boat (see figure). Let θ be the angle of elevation from the boat to the winch and let s be the length of the rope from the winch to the boat.



(a) Write θ as a function of s.

(b) Find θ when $s = 40$ feet and $s = 20$ feet.

106. PHOTOGRAPHY A television camera at ground level is filming the lift-off of a space shuttle at a point 750 meters from the launch pad (see figure). Let θ be the angle of elevation to the shuttle and let s be the height of the shuttle.

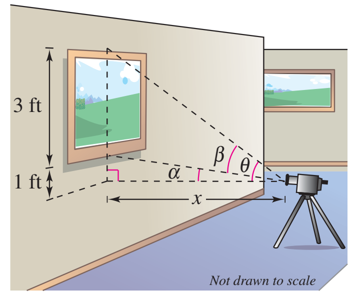


(a) Write θ as a function of s.

(b) Find θ when $s = 300$ meters and $s = 1200$ meters.

107. PHOTOGRAPHY A photographer is taking a picture of a three-foot-tall painting hung in an art gallery. The camera lens is 1 foot below the lower edge of the painting (see figure). The angle β subtended by the camera lens x feet from the painting is

$$\beta = \arctan \frac{3x}{x^2 + 4}, \quad x > 0.$$

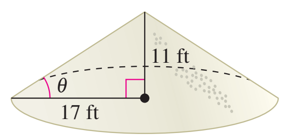


(a) Use a graphing utility to graph β as a function of x.

(b) Move the cursor along the graph to approximate the distance from the picture when β is maximum.

(c) Identify the asymptote of the graph and discuss its meaning in the context of the problem.

108. GRANULAR ANGLE OF REPOSE Different types of granular substances naturally settle at different angles when stored in cone-shaped piles. This angle θ is called the *angle of repose* (see figure). When rock salt is stored in a cone-shaped pile 11 feet high, the diameter of the pile's base is about 34 feet. (Source: Bulk-Store Structures, Inc.)



(a) Find the angle of repose for rock salt.

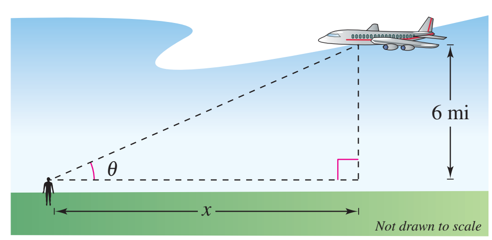
(b) How tall is a pile of rock salt that has a base diameter of 40 feet?

109. GRANULAR ANGLE OF REPOSE When whole corn is stored in a cone-shaped pile 20 feet high, the diameter of the pile's base is about 82 feet.

(a) Find the angle of repose for whole corn.

(b) How tall is a pile of corn that has a base diameter of 100 feet?

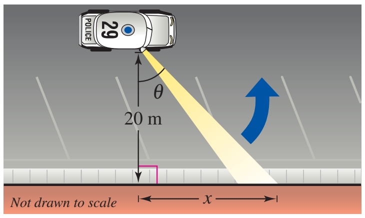
110. ANGLE OF ELEVATION An airplane flies at an altitude of 6 miles toward a point directly over an observer. Consider θ and x as shown in the figure.



(a) Write θ as a function of x.

(b) Find θ when $x = 7$ miles and $x = 1$ mile.

111. SECURITY PATROL A security car with its spotlight on is parked 20 meters from a warehouse. Consider θ and x as shown in the figure.



(a) Write θ as a function of x.

(b) Find θ when $x = 5$ meters and $x = 12$ meters.

EXPLORATION

TRUE OR FALSE? In Exercises 112–114, determine whether the statement is true or false. Justify your answer.

112. $\sin \frac{5\pi}{6} = \frac{1}{2}$ ⇨ $\arcsin \frac{1}{2} = \frac{5\pi}{6}$

113. $\tan \frac{5\pi}{4} = 1$ ⇨ $\arctan 1 = \frac{5\pi}{4}$

114. $\arctan x = \dfrac{\arcsin x}{\arccos x}$

115. Define the inverse cotangent function by restricting the domain of the cotangent function to the interval $(0, \pi)$, and sketch its graph.

116. Define the inverse secant function by restricting the domain of the secant function to the intervals $[0, \pi/2)$ and $(\pi/2, \pi]$, and sketch its graph.

117. Define the inverse cosecant function by restricting the domain of the cosecant function to the intervals $[-\pi/2, 0)$ and $(0, \pi/2]$, and sketch its graph.

118. CAPSTONE Use the results of Exercises 115–117 to explain how to graph (a) the inverse cotangent function, (b) the inverse secant function, and (c) the inverse cosecant function on a graphing utility.

In Exercises 119–126, use the results of Exercises 115–117 to evaluate each expression without using a calculator.

119. $\operatorname{arcsec} \sqrt{2}$ **120.** $\operatorname{arcsec} 1$

121. $\operatorname{arccot}(-1)$ **122.** $\operatorname{arccot}(-\sqrt{3})$

123. $\operatorname{arccsc} 2$ **124.** $\operatorname{arccsc}(-1)$

125. $\operatorname{arccsc}\left(\frac{2\sqrt{3}}{3}\right)$ **126.** $\operatorname{arcsec}\left(-\frac{2\sqrt{3}}{3}\right)$

In Exercises 127–134, use the results of Exercises 115–117 and a calculator to approximate the value of the expression. Round your result to two decimal places.

127. $\operatorname{arcsec} 2.54$ **128.** $\operatorname{arcsec}(-1.52)$

129. $\operatorname{arccot} 5.25$ **130.** $\operatorname{arccot}(-10)$

131. $\operatorname{arccot} \frac{5}{3}$ **132.** $\operatorname{arccot}\left(-\frac{16}{7}\right)$

133. $\operatorname{arccsc}\left(-\frac{25}{3}\right)$ **134.** $\operatorname{arccsc}(-12)$

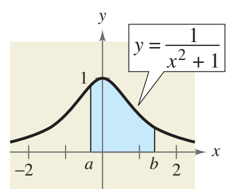
135. AREA In calculus, it is shown that the area of the region bounded by the graphs of $y = 0$, $y = 1/(x^2 + 1)$, $x = a$, and $x = b$ is given by

Area $= \arctan b - \arctan a$

(see figure). Find the area for the following values of a and b.

(a) $a = 0, b = 1$ (b) $a = -1, b = 1$

(c) $a = 0, b = 3$ (d) $a = -1, b = 3$



136. THINK ABOUT IT Use a graphing utility to graph the functions

$f(x) = \sqrt{x}$ and $g(x) = 6 \arctan x$.

For $x > 0$, it appears that $g > f$. Explain why you know that there exists a positive real number a such that $g < f$ for $x > a$. Approximate the number a.

137. THINK ABOUT IT Consider the functions given by

$f(x) = \sin x$ and $f^{-1}(x) = \arcsin x$.

(a) Use a graphing utility to graph the composite functions $f \circ f^{-1}$ and $f^{-1} \circ f$.

(b) Explain why the graphs in part (a) are not the graph of the line $y = x$. Why do the graphs of $f \circ f^{-1}$ and $f^{-1} \circ f$ differ?

138. PROOF Prove each identity.

(a) $\arcsin(-x) = -\arcsin x$

(b) $\arctan(-x) = -\arctan x$

(c) $\arctan x + \arctan \frac{1}{x} = \frac{\pi}{2}, \quad x > 0$

(d) $\arcsin x + \arccos x = \frac{\pi}{2}$

(e) $\arcsin x = \arctan \dfrac{x}{\sqrt{1 - x^2}}$