4.6 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

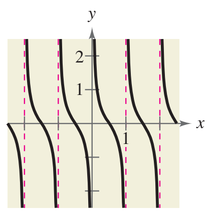
VOCABULARY: Fill in the blanks.

1. The tangent, cotangent, and cosecant functions are ________ , so the graphs of these functions have symmetry with respect to the ________.
2. The graphs of the tangent, cotangent, secant, and cosecant functions all have ________ asymptotes.
3. To sketch the graph of a secant or cosecant function, first make a sketch of its corresponding ________ function.
4. For the functions given by $f(x) = g(x) \cdot \sin x$, $g(x)$ is called the ________ factor of the function $f(x)$.
5. The period of $y = \tan x$ is ________.
6. The domain of $y = \cot x$ is all real numbers such that ________.
7. The range of $y = \sec x$ is ________.
8. The period of $y = \csc x$ is ________.

SKILLS AND APPLICATIONS

In Exercises 9–14, match the function with its graph. State the period of the function. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

(a)



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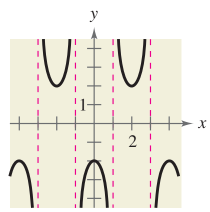

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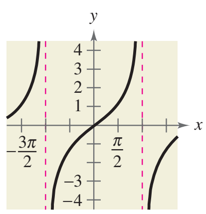

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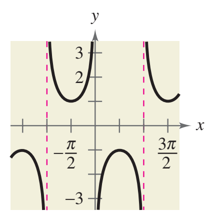

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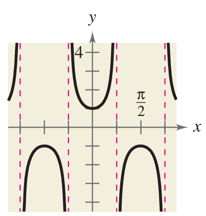
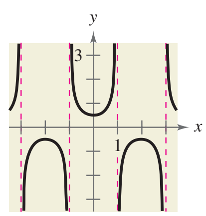


(f)



9. $y = \sec 2x$
10. $y = \tan \frac{x}{2}$
11. $y = \frac{1}{2} \cot \pi x$
12. $y = -\csc x$
13. $y = \frac{1}{2} \sec \frac{\pi x}{2}$
14. $y = -2 \sec \frac{\pi x}{2}$

In Exercises 15–38, sketch the graph of the function. Include two full periods.

15. $y = \frac{1}{3} \tan x$
16. $y = \tan 4x$
17. $y = -2 \tan 3x$
18. $y = -3 \tan \pi x$
19. $y = -\frac{1}{2} \sec x$
20. $y = \frac{1}{4} \sec x$
21. $y = \csc \pi x$
22. $y = 3 \csc 4x$
23. $y = \frac{1}{2} \sec \pi x$
24. $y = -2 \sec 4x + 2$
25. $y = \csc \frac{x}{2}$
26. $y = \csc \frac{x}{3}$
27. $y = 3 \cot 2x$
28. $y = 3 \cot \frac{\pi x}{2}$
29. $y = 2 \sec 3x$
30. $y = -\frac{1}{2} \tan x$
31. $y = \tan \frac{\pi x}{4}$
32. $y = \tan(x + \pi)$
33. $y = 2 \csc(x - \pi)$
34. $y = \csc(2x - \pi)$
35. $y = 2 \sec(x + \pi)$
36. $y = -\sec \pi x + 1$
37. $y = \frac{1}{4} \csc\left(x + \frac{\pi}{4}\right)$
38. $y = 2 \cot\left(x + \frac{\pi}{2}\right)$

In Exercises 39–48, use a graphing utility to graph the function. Include two full periods.

39. $y = \tan \frac{x}{3}$
40. $y = -\tan 2x$
41. $y = -2 \sec 4x$
42. $y = \sec \pi x$
43. $y = \tan\left(x - \frac{\pi}{4}\right)$
44. $y = \frac{1}{4} \cot\left(x - \frac{\pi}{2}\right)$
45. $y = -\csc(4x - \pi)$
46. $y = 2 \sec(2x - \pi)$
47. $y = 0.1 \tan\left(\frac{\pi x}{4} + \frac{\pi}{4}\right)$
48. $y = \frac{1}{3} \sec\left(\frac{\pi x}{2} + \frac{\pi}{2}\right)$

In Exercises 49–56, use a graph to solve the equation on the interval $[-2\pi, 2\pi]$.

49. $\tan x = 1$

50. $\tan x = \sqrt{3}$

51. $\cot x = -\dfrac{\sqrt{3}}{3}$

52. $\cot x = 1$

53. $\sec x = -2$

54. $\sec x = 2$

55. $\csc x = \sqrt{2}$

56. $\csc x = -\dfrac{2\sqrt{3}}{3}$

In Exercises 57–64, use the graph of the function to determine whether the function is even, odd, or neither. Verify your answer algebraically.

57. $f(x) = \sec x$

58. $f(x) = \tan x$

59. $g(x) = \cot x$

60. $g(x) = \csc x$

61. $f(x) = x + \tan x$

62. $f(x) = x^2 - \sec x$

63. $g(x) = x \csc x$

64. $g(x) = x^2 \cot x$

65. GRAPHICAL REASONING Consider the functions given by

$$f(x) = 2 \sin x \quad \text{and} \quad g(x) = \frac{1}{2} \csc x$$

on the interval $(0, \pi)$.

(a) Graph f and g in the same coordinate plane.

(b) Approximate the interval in which $f > g$.

(c) Describe the behavior of each of the functions as x approaches π. How is the behavior of g related to the behavior of f as x approaches π?

66. GRAPHICAL REASONING Consider the functions given by

$$f(x) = \tan \frac{\pi x}{2} \quad \text{and} \quad g(x) = \frac{1}{2} \sec \frac{\pi x}{2}$$

on the interval $(-1, 1)$.

(a) Use a graphing utility to graph f and g in the same viewing window.

(b) Approximate the interval in which $f < g$.

(c) Approximate the interval in which $2f < 2g$. How does the result compare with that of part (b)? Explain.

In Exercises 67–72, use a graphing utility to graph the two equations in the same viewing window. Use the graphs to determine whether the expressions are equivalent. Verify the results algebraically.

67. $y_1 = \sin x \csc x, \quad y_2 = 1$

68. $y_1 = \sin x \sec x, \quad y_2 = \tan x$

69. $y_1 = \dfrac{\cos x}{\sin x}, \quad y_2 = \cot x$

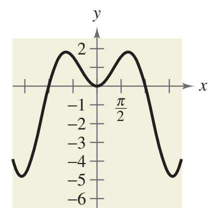
70. $y_1 = \tan x \cot^2 x, \quad y_2 = \cot x$

71. $y_1 = 1 + \cot^2 x, \quad y_2 = \csc^2 x$

72. $y_1 = \sec^2 x - 1, \quad y_2 = \tan^2 x$

In Exercises 73–76, match the function with its graph. Describe the behavior of the function as x approaches zero. [The graphs are labeled (a), (b), (c), and (d).]

(a)



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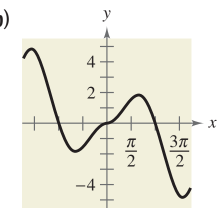

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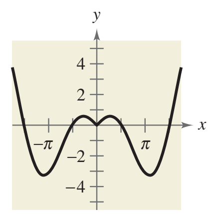
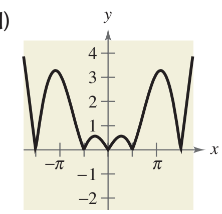


(d)



73. $f(x) = |x \cos x|$

74. $f(x) = x \sin x$

75. $g(x) = |x| \sin x$

76. $g(x) = |x| \cos x$

CONJECTURE In Exercises 77–80, graph the functions f and g. Use the graphs to make a conjecture about the relationship between the functions.

77. $f(x) = \sin x + \cos\left(x + \dfrac{\pi}{2}\right), \quad g(x) = 0$

78. $f(x) = \sin x - \cos\left(x + \dfrac{\pi}{2}\right), \quad g(x) = 2 \sin x$

79. $f(x) = \sin^2 x, \quad g(x) = \frac{1}{2}(1 - \cos 2x)$

80. $f(x) = \cos^2 \dfrac{\pi x}{2}, \quad g(x) = \dfrac{1}{2}(1 + \cos \pi x)$

In Exercises 81–84, use a graphing utility to graph the function and the damping factor of the function in the same viewing window. Describe the behavior of the function as x increases without bound.

81. $g(x) = e^{-x^2/2} \sin x$

82. $f(x) = e^{-x} \cos x$

83. $f(x) = 2^{-x/4} \cos \pi x$

84. $h(x) = 2^{-x^2/4} \sin x$

In Exercises 85–90, use a graphing utility to graph the function. Describe the behavior of the function as x approaches zero.

85. $y = \dfrac{6}{x} + \cos x, \quad x > 0$

86. $y = \dfrac{4}{x} + \sin 2x, \quad x > 0$

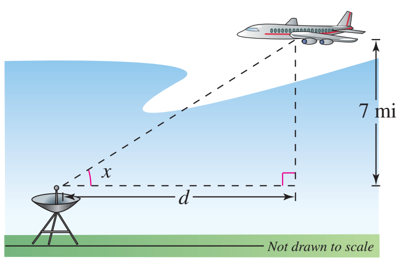
87. $g(x) = \dfrac{\sin x}{x}$

88. $f(x) = \dfrac{1 - \cos x}{x}$

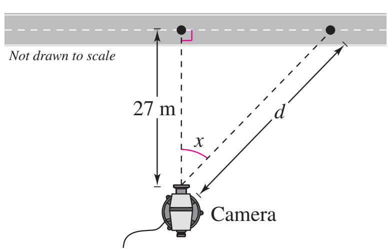
89. $f(x) = \sin \dfrac{1}{x}$

90. $h(x) = x \sin \dfrac{1}{x}$

91. DISTANCE A plane flying at an altitude of 7 miles above a radar antenna will pass directly over the radar antenna (see figure). Let d be the ground distance from the antenna to the point directly under the plane and let x be the angle of elevation to the plane from the antenna. (d is positive as the plane approaches the antenna.) Write d as a function of x and graph the function over the interval $0 < x < \pi$.



92. TELEVISION COVERAGE A television camera is on a reviewing platform 27 meters from the street on which a parade will be passing from left to right (see figure). Write the distance d from the camera to a particular unit in the parade as a function of the angle x, and graph the function over the interval $-\pi/2 < x < \pi/2$. (Consider x as negative when a unit in the parade approaches from the left.)



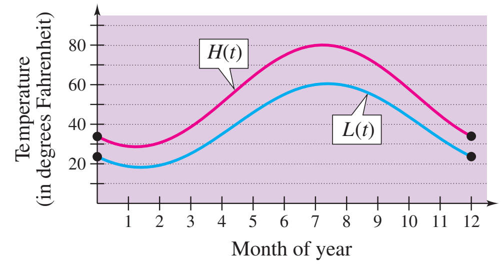
93. METEOROLOGY The normal monthly high temperatures H (in degrees Fahrenheit) in Erie, Pennsylvania are approximated by

$$H(t) = 56.94 - 20.86 \cos(\pi t/6) - 11.58 \sin(\pi t/6)$$

and the normal monthly low temperatures L are approximated by

$$L(t) = 41.80 - 17.13 \cos(\pi t/6) - 13.39 \sin(\pi t/6)$$

where t is the time (in months), with $t = 1$ corresponding to January (see figure). (Source: National Climatic Data Center)



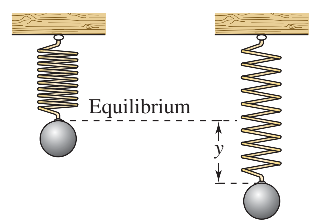
(a) What is the period of each function?

(b) During what part of the year is the difference between the normal high and normal low temperatures greatest? When is it smallest?

(c) The sun is northernmost in the sky around June 21, but the graph shows the warmest temperatures at a later date. Approximate the lag time of the temperatures relative to the position of the sun.

94. SALES The projected monthly sales S (in thousands of units) of lawn mowers (a seasonal product) are modeled by $S = 74 + 3t - 40 \cos(\pi t/6)$, where t is the time (in months), with $t = 1$ corresponding to January. Graph the sales function over 1 year.

95. HARMONIC MOTION An object weighing W pounds is suspended from the ceiling by a steel spring (see figure). The weight is pulled downward (positive direction) from its equilibrium position and released. The resulting motion of the weight is described by the function $y = \frac{1}{2}e^{-t/4} \cos 4t$, $t > 0$, where y is the distance (in feet) and t is the time (in seconds).



(a) Use a graphing utility to graph the function.

(b) Describe the behavior of the displacement function for increasing values of time t.

EXPLORATION

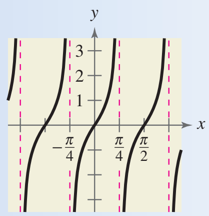
TRUE OR FALSE? In Exercises 96 and 97, determine whether the statement is true or false. Justify your answer.

96. The graph of $y = \csc x$ can be obtained on a calculator by graphing the reciprocal of $y = \sin x$.

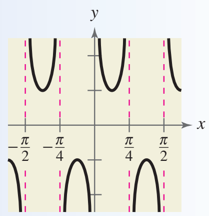
97. The graph of $y = \sec x$ can be obtained on a calculator by graphing a translation of the reciprocal of $y = \sin x$.

98. CAPSTONE Determine which function is represented by the graph. Do not use a calculator. Explain your reasoning.

(a)



(b)



(a)	(b)
(i) $f(x) = \tan 2x$	(i) $f(x) = \sec 4x$
(ii) $f(x) = \tan(x/2)$	(ii) $f(x) = \csc 4x$
(iii) $f(x) = 2 \tan x$	(iii) $f(x) = \csc(x/4)$
(iv) $f(x) = -\tan 2x$	(iv) $f(x) = \sec(x/4)$
(v) $f(x) = -\tan(x/2)$	(v) $f(x) = \csc(4x - \pi)$

In Exercises 99 and 100, use a graphing utility to graph the function. Use the graph to determine the behavior of the function as $x \to c$.

(a) $x \to \frac{\pi^+}{2}$ $\left(\text{as } x \text{ approaches } \frac{\pi}{2} \text{ from the right}\right)$

(b) $x \to \frac{\pi^-}{2}$ $\left(\text{as } x \text{ approaches } \frac{\pi}{2} \text{ from the left}\right)$

(c) $x \to -\frac{\pi^+}{2}$ $\left(\text{as } x \text{ approaches } -\frac{\pi}{2} \text{ from the right}\right)$

(d) $x \to -\frac{\pi^-}{2}$ $\left(\text{as } x \text{ approaches } -\frac{\pi}{2} \text{ from the left}\right)$

99. $f(x) = \tan x$ **100.** $f(x) = \sec x$

In Exercises 101 and 102, use a graphing utility to graph the function. Use the graph to determine the behavior of the function as $x \to c$.

(a) As $x \to 0^+$, the value of $f(x) \to$ ☐.

(b) As $x \to 0^-$, the value of $f(x) \to$ ☐.

(c) As $x \to \pi^+$, the value of $f(x) \to$ ☐.

(d) As $x \to \pi^-$, the value of $f(x) \to$ ☐.

101. $f(x) = \cot x$ **102.** $f(x) = \csc x$

103. THINK ABOUT IT Consider the function given by $f(x) = x - \cos x$.

(a) Use a graphing utility to graph the function and verify that there exists a zero between 0 and 1. Use the graph to approximate the zero.

(b) Starting with $x_0 = 1$, generate a sequence $x_1, x_2, x_3, \ldots$, where $x_n = \cos(x_{n-1})$. For example,

$$x_0 = 1$$
$$x_1 = \cos(x_0)$$
$$x_2 = \cos(x_1)$$
$$x_3 = \cos(x_2)$$
$$\vdots$$

What value does the sequence approach?

104. APPROXIMATION Using calculus, it can be shown that the tangent function can be approximated by the polynomial

$$\tan x \approx x + \frac{2x^3}{3!} + \frac{16x^5}{5!}$$

where x is in radians. Use a graphing utility to graph the tangent function and its polynomial approximation in the same viewing window. How do the graphs compare?

105. APPROXIMATION Using calculus, it can be shown that the secant function can be approximated by the polynomial

$$\sec x \approx 1 + \frac{x^2}{2!} + \frac{5x^4}{4!}$$

where x is in radians. Use a graphing utility to graph the secant function and its polynomial approximation in the same viewing window. How do the graphs compare?

106. PATTERN RECOGNITION

(a) Use a graphing utility to graph each function.

$$y_1 = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x\right)$$

$$y_2 = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x\right)$$

(b) Identify the pattern started in part (a) and find a function y_3 that continues the pattern one more term. Use a graphing utility to graph y_3.

(c) The graphs in parts (a) and (b) approximate the periodic function in the figure. Find a function y_4 that is a better approximation.