4.5 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

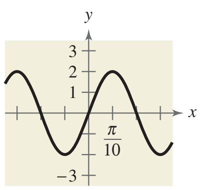
VOCABULARY: Fill in the blanks.

1. One period of a sine or cosine function is called one ________ of the sine or cosine curve.
2. The ________ of a sine or cosine curve represents half the distance between the maximum and minimum values of the function.
3. For the function given by $y = a\sin(bx - c)$, $\frac{c}{b}$ represents the ________ ________ of the graph of the function.
4. For the function given by $y = d + a\cos(bx - c)$, d represents a ________ ________ of the graph of the function.

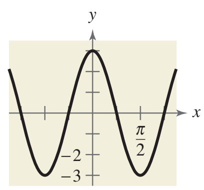
SKILLS AND APPLICATIONS

In Exercises 5–18, find the period and amplitude.

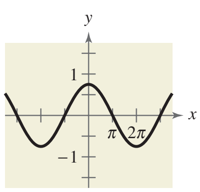
5. $y = 2\sin 5x$



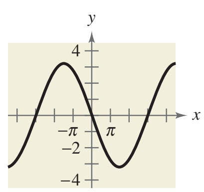
6. $y = 3\cos 2x$



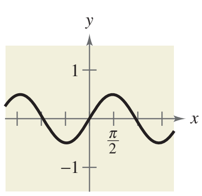
7. $y = \frac{3}{4}\cos\frac{x}{2}$



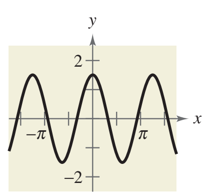
8. $y = -3\sin\frac{x}{3}$



9. $y = \frac{1}{2}\sin\frac{\pi x}{3}$



10. $y = \frac{3}{2}\cos\frac{\pi x}{2}$



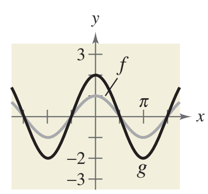
11. $y = -4\sin x$
12. $y = -\cos\frac{2x}{3}$
13. $y = 3\sin 10x$
14. $y = \frac{1}{5}\sin 6x$
15. $y = \frac{5}{3}\cos\frac{4x}{5}$
16. $y = \frac{5}{2}\cos\frac{x}{4}$
17. $y = \frac{1}{4}\sin 2\pi x$
18. $y = \frac{2}{3}\cos\frac{\pi x}{10}$

In Exercises 19–26, describe the relationship between the graphs of f and g. Consider amplitude, period, and shifts.

19. $f(x) = \sin x$
    $g(x) = \sin(x - \pi)$
20. $f(x) = \cos x$
    $g(x) = \cos(x + \pi)$
21. $f(x) = \cos 2x$
    $g(x) = -\cos 2x$
22. $f(x) = \sin 3x$
    $g(x) = \sin(-3x)$
23. $f(x) = \cos x$
    $g(x) = \cos 2x$
24. $f(x) = \sin x$
    $g(x) = \sin 3x$
25. $f(x) = \sin 2x$
    $g(x) = 3 + \sin 2x$
26. $f(x) = \cos 4x$
    $g(x) = -2 + \cos 4x$

In Exercises 27–30, describe the relationship between the graphs of f and g. Consider amplitude, period, and shifts.

27.



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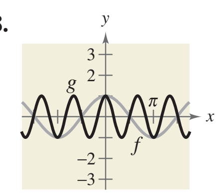

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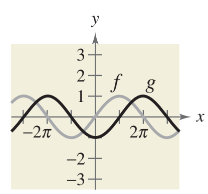

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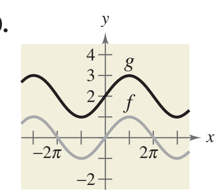


In Exercises 31–38, graph f and g on the same set of coordinate axes. (Include two full periods.)

31. $f(x) = -2\sin x$
    $g(x) = 4\sin x$
32. $f(x) = \sin x$
    $g(x) = \sin\frac{x}{3}$
33. $f(x) = \cos x$
    $g(x) = 2 + \cos x$
34. $f(x) = 2\cos 2x$
    $g(x) = -\cos 4x$

35. $f(x) = -\frac{1}{2}\sin\frac{x}{2}$

$g(x) = 3 - \frac{1}{2}\sin\frac{x}{2}$

36. $f(x) = 4\sin \pi x$

$g(x) = 4\sin \pi x - 3$

37. $f(x) = 2\cos x$

$g(x) = 2\cos(x + \pi)$

38. $f(x) = -\cos x$

$g(x) = -\cos(x - \pi)$

In Exercises 39–60, sketch the graph of the function. (Include two full periods.)

39. $y = 5\sin x$ **40.** $y = \frac{1}{4}\sin x$

41. $y = \frac{1}{3}\cos x$ **42.** $y = 4\cos x$

43. $y = \cos\frac{x}{2}$ **44.** $y = \sin 4x$

45. $y = \cos 2\pi x$ **46.** $y = \sin\frac{\pi x}{4}$

47. $y = -\sin\frac{2\pi x}{3}$ **48.** $y = -10\cos\frac{\pi x}{6}$

49. $y = \sin\left(x - \frac{\pi}{2}\right)$ **50.** $y = \sin(x - 2\pi)$

51. $y = 3\cos(x + \pi)$ **52.** $y = 4\cos\left(x + \frac{\pi}{4}\right)$

53. $y = 2 - \sin\frac{2\pi x}{3}$ **54.** $y = -3 + 5\cos\frac{\pi t}{12}$

55. $y = 2 + \frac{1}{10}\cos 60\pi x$ **56.** $y = 2\cos x - 3$

57. $y = 3\cos(x + \pi) - 3$ **58.** $y = 4\cos\left(x + \frac{\pi}{4}\right) + 4$

59. $y = \frac{2}{3}\cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$ **60.** $y = -3\cos(6x + \pi)$

In Exercises 61–66, g is related to a parent function $f(x) = \sin(x)$ or $f(x) = \cos(x)$. (a) Describe the sequence of transformations from f to g. (b) Sketch the graph of g. (c) Use function notation to write g in terms of f.

61. $g(x) = \sin(4x - \pi)$ **62.** $g(x) = \sin(2x + \pi)$

63. $g(x) = \cos(x - \pi) + 2$ **64.** $g(x) = 1 + \cos(x + \pi)$

65. $g(x) = 2\sin(4x - \pi) - 3$ **66.** $g(x) = 4 - \sin(2x + \pi)$

In Exercises 67–72, use a graphing utility to graph the function. Include two full periods. Be sure to choose an appropriate viewing window.

67. $y = -2\sin(4x + \pi)$ **68.** $y = -4\sin\left(\frac{2}{3}x - \frac{\pi}{3}\right)$

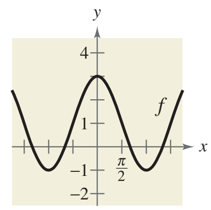
69. $y = \cos\left(2\pi x - \frac{\pi}{2}\right) + 1$

70. $y = 3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) - 2$

71. $y = -0.1\sin\left(\frac{\pi x}{10} + \pi\right)$ **72.** $y = \frac{1}{100}\sin 120\pi t$

GRAPHICAL REASONING In Exercises 73–76, find a and d for the function $f(x) = a\cos x + d$ such that the graph of f matches the figure.

73.



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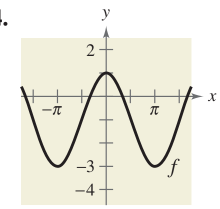

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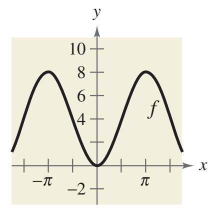

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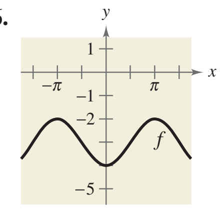
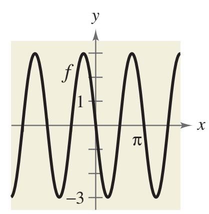


GRAPHICAL REASONING In Exercises 77–80, find a, b, and c for the function $f(x) = a\sin(bx - c)$ such that the graph of f matches the figure.

77.



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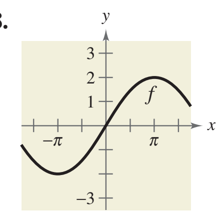

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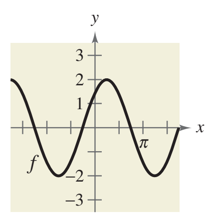
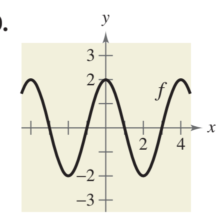


80.



In Exercises 81 and 82, use a graphing utility to graph y_1 and y_2 in the interval $[-2\pi, 2\pi]$. Use the graphs to find real numbers x such that $y_1 = y_2$.

81. $y_1 = \sin x$

$y_2 = -\frac{1}{2}$

82. $y_1 = \cos x$

$y_2 = -1$

In Exercises 83–86, write an equation for the function that is described by the given characteristics.

83. A sine curve with a period of π, an amplitude of 2, a right phase shift of $\pi/2$, and a vertical translation up 1 unit

84. A sine curve with a period of 4π, an amplitude of 3, a left phase shift of $\pi/4$, and a vertical translation down 1 unit

85. A cosine curve with a period of π, an amplitude of 1, a left phase shift of π, and a vertical translation down $\frac{3}{2}$ units

86. A cosine curve with a period of 4π, an amplitude of 3, a right phase shift of $\pi/2$, and a vertical translation up 2 units

87. RESPIRATORY CYCLE For a person at rest, the velocity v (in liters per second) of airflow during a respiratory cycle (the time from the beginning of one breath to the beginning of the next) is given by $v = 0.85 \sin \frac{\pi t}{3}$, where t is the time (in seconds). (Inhalation occurs when $v > 0$, and exhalation occurs when $v < 0$.)

(a) Find the time for one full respiratory cycle.

(b) Find the number of cycles per minute.

(c) Sketch the graph of the velocity function.

88. RESPIRATORY CYCLE After exercising for a few minutes, a person has a respiratory cycle for which the velocity of airflow is approximated by $v = 1.75 \sin \frac{\pi t}{2}$, where t is the time (in seconds). (Inhalation occurs when $v > 0$, and exhalation occurs when $v < 0$.)

(a) Find the time for one full respiratory cycle.

(b) Find the number of cycles per minute.

(c) Sketch the graph of the velocity function.

89. DATA ANALYSIS: METEOROLOGY The table shows the maximum daily high temperatures in Las Vegas L and International Falls I (in degrees Fahrenheit) for month t, with $t = 1$ corresponding to January. (Source: National Climatic Data Center)

Month, t	Las Vegas, L	International Falls, I
1	57.1	13.8
2	63.0	22.4
3	69.5	34.9
4	78.1	51.5
5	87.8	66.6
6	98.9	74.2
7	104.1	78.6
8	101.8	76.3
9	93.8	64.7
10	80.8	51.7
11	66.0	32.5
12	57.3	18.1

(a) A model for the temperature in Las Vegas is given by

$$L(t) = 80.60 + 23.50 \cos\left(\frac{\pi t}{6} - 3.67\right).$$

Find a trigonometric model for International Falls.

(b) Use a graphing utility to graph the data points and the model for the temperatures in Las Vegas. How well does the model fit the data?

(c) Use a graphing utility to graph the data points and the model for the temperatures in International Falls. How well does the model fit the data?

(d) Use the models to estimate the average maximum temperature in each city. Which term of the models did you use? Explain.

(e) What is the period of each model? Are the periods what you expected? Explain.

(f) Which city has the greater variability in temperature throughout the year? Which factor of the models determines this variability? Explain.

90. HEALTH The function given by

$$P = 100 - 20 \cos \frac{5\pi t}{3}$$

approximates the blood pressure P (in millimeters of mercury) at time t (in seconds) for a person at rest.

(a) Find the period of the function.

(b) Find the number of heartbeats per minute.

91. PIANO TUNING When tuning a piano, a technician strikes a tuning fork for the A above middle C and sets up a wave motion that can be approximated by $y = 0.001 \sin 880\pi t$, where t is the time (in seconds).

(a) What is the period of the function?

(b) The frequency f is given by $f = 1/p$. What is the frequency of the note?

92. DATA ANALYSIS: ASTRONOMY The percents y (in decimal form) of the moon's face that was illuminated on day x in the year 2009, where $x = 1$ represents January 1, are shown in the table. (Source: U.S. Naval Observatory)

x	y
4	0.5
11	1.0
18	0.5
26	0.0
33	0.5
40	1.0