4.4 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

In Exercises 1–6, let θ be an angle in standard position, with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

1. $\sin\theta =$ _____ **2.** $\frac{r}{y} =$ _____

3. $\tan\theta =$ _____ **4.** $\sec\theta =$ _____

5. $\frac{x}{r} =$ _____ **6.** $\frac{x}{y} =$ _____

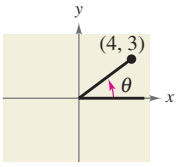
7. Because $r = \sqrt{x^2 + y^2}$ cannot be _____, the sine and cosine functions are _____ for any real value of θ.

8. The acute positive angle that is formed by the terminal side of the angle θ and the horizontal axis is called the _____ angle of θ and is denoted by θ'.

SKILLS AND APPLICATIONS

In Exercises 9–12, determine the exact values of the six trigonometric functions of the angle θ.

9. (a)



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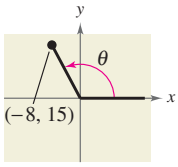
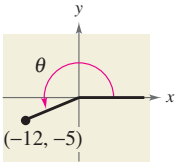


10. (a)



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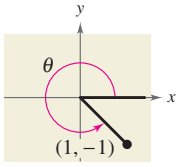
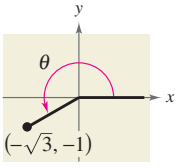


11. (a)



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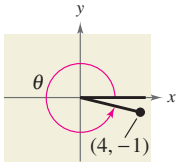
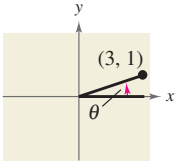


12. (a)



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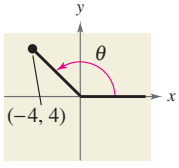


In Exercises 13–18, the point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

13. $(5, 12)$ **14.** $(8, 15)$

15. $(-5, -2)$ **16.** $(-4, 10)$

17. $(-5.4, 7.2)$ **18.** $\left(3\frac{1}{2}, -7\frac{3}{4}\right)$

In Exercises 19–22, state the quadrant in which θ lies.

19. $\sin\theta > 0$ and $\cos\theta > 0$

20. $\sin\theta < 0$ and $\cos\theta < 0$

21. $\sin\theta > 0$ and $\cos\theta < 0$

22. $\sec\theta > 0$ and $\cot\theta < 0$

In Exercises 23–32, find the values of the six trigonometric functions of θ with the given constraint.

Function Value	*Constraint*
23. $\tan\theta = -\frac{15}{8}$	$\sin\theta > 0$
24. $\cos\theta = \frac{8}{17}$	$\tan\theta < 0$
25. $\sin\theta = \frac{3}{5}$	θ lies in Quadrant II.
26. $\cos\theta = -\frac{4}{5}$	θ lies in Quadrant III.
27. $\cot\theta = -3$	$\cos\theta > 0$
28. $\csc\theta = 4$	$\cot\theta < 0$
29. $\sec\theta = -2$	$\sin\theta < 0$
30. $\sin\theta = 0$	$\sec\theta = -1$
31. $\cot\theta$ is undefined.	$\pi/2 \leq \theta \leq 3\pi/2$
32. $\tan\theta$ is undefined.	$\pi \leq \theta \leq 2\pi$

In Exercises 33–36, the terminal side of θ lies on the given line in the specified quadrant. Find the values of the six trigonometric functions of θ by finding a point on the line.

Line	*Quadrant*
33. $y = -x$	II
34. $y = \frac{1}{3}x$	III
35. $2x - y = 0$	III
36. $4x + 3y = 0$	IV

In Exercises 37–44, evaluate the trigonometric function of the quadrant angle.

37. $\sin \pi$

38. $\csc \frac{3\pi}{2}$

39. $\sec \frac{3\pi}{2}$

40. $\sec \pi$

41. $\sin \frac{\pi}{2}$

42. $\cot \pi$

43. $\csc \pi$

44. $\cot \frac{\pi}{2}$

In Exercises 45–52, find the reference angle θ', and sketch θ and θ' in standard position.

45. $\theta = 160°$

46. $\theta = 309°$

47. $\theta = -125°$

48. $\theta = -215°$

49. $\theta = \frac{2\pi}{3}$

50. $\theta = \frac{7\pi}{6}$

51. $\theta = 4.8$

52. $\theta = 11.6$

In Exercises 53–68, evaluate the sine, cosine, and tangent of the angle without using a calculator.

53. $225°$

54. $300°$

55. $750°$

56. $-405°$

57. $-150°$

58. $-840°$

59. $\frac{2\pi}{3}$

60. $\frac{3\pi}{4}$

61. $\frac{5\pi}{4}$

62. $\frac{7\pi}{6}$

63. $-\frac{\pi}{6}$

64. $-\frac{\pi}{2}$

65. $\frac{9\pi}{4}$

66. $\frac{10\pi}{3}$

67. $-\frac{3\pi}{2}$

68. $-\frac{23\pi}{4}$

In Exercises 69–74, find the indicated trigonometric value in the specified quadrant.

	Function	*Quadrant*	*Trigonometric Value*
69.	$\sin \theta = -\frac{3}{5}$	IV	$\cos \theta$
70.	$\cot \theta = -3$	II	$\sin \theta$
71.	$\tan \theta = \frac{3}{2}$	III	$\sec \theta$
72.	$\csc \theta = -2$	IV	$\cot \theta$
73.	$\cos \theta = \frac{5}{8}$	I	$\sec \theta$
74.	$\sec \theta = -\frac{9}{4}$	III	$\tan \theta$

In Exercises 75–90, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is set in the correct angle mode.)

75. $\sin 10°$

76. $\sec 225°$

77. $\cos(-110°)$

78. $\csc(-330°)$

79. $\tan 304°$

80. $\cot 178°$

81. $\sec 72°$

82. $\tan(-188°)$

83. $\tan 4.5$

84. $\cot 1.35$

85. $\tan \frac{\pi}{9}$

86. $\tan\left(-\frac{\pi}{9}\right)$

87. $\sin(-0.65)$

88. $\sec 0.29$

89. $\cot\left(-\frac{11\pi}{8}\right)$

90. $\csc\left(-\frac{15\pi}{14}\right)$

In Exercises 91–96, find two solutions of the equation. Give your answers in degrees ($0° \le \theta < 360°$) and in radians ($0 \le \theta < 2\pi$). Do not use a calculator.

91. (a) $\sin \theta = \frac{1}{2}$ (b) $\sin \theta = -\frac{1}{2}$

92. (a) $\cos \theta = \frac{\sqrt{2}}{2}$ (b) $\cos \theta = -\frac{\sqrt{2}}{2}$

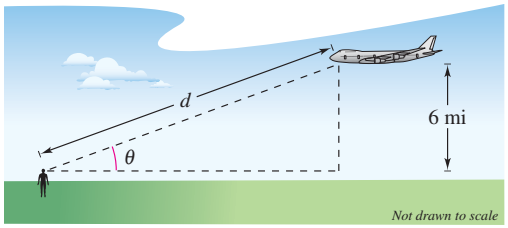
93. (a) $\csc \theta = \frac{2\sqrt{3}}{3}$ (b) $\cot \theta = -1$

94. (a) $\sec \theta = 2$ (b) $\sec \theta = -2$

95. (a) $\tan \theta = 1$ (b) $\cot \theta = -\sqrt{3}$

96. (a) $\sin \theta = \frac{\sqrt{3}}{2}$ (b) $\sin \theta = -\frac{\sqrt{3}}{2}$

97. DISTANCE An airplane, flying at an altitude of 6 miles, is on a flight path that passes directly over an observer (see figure). If θ is the angle of elevation from the observer to the plane, find the distance d from the observer to the plane when (a) $\theta = 30°$, (b) $\theta = 90°$, and (c) $\theta = 120°$.



98. HARMONIC MOTION The displacement from equilibrium of an oscillating weight suspended by a spring is given by $y(t) = 2 \cos 6t$, where y is the displacement (in centimeters) and t is the time (in seconds). Find the displacement when (a) $t = 0$, (b) $t = \frac{1}{4}$, and (c) $t = \frac{1}{2}$.

99. DATA ANALYSIS: METEOROLOGY The table shows the monthly normal temperatures (in degrees Fahrenheit) for selected months in New York City (N) and Fairbanks, Alaska (F). (Source: National Climatic Data Center)

Month	New York City, N	Fairbanks, F
January	33	-10
April	52	32
July	77	62
October	58	24
December	38	-6

(a) Use the *regression* feature of a graphing utility to find a model of the form $y = a\sin(bt + c) + d$ for each city. Let t represent the month, with $t = 1$ corresponding to January.

(b) Use the models from part (a) to find the monthly normal temperatures for the two cities in February, March, May, June, August, September, and November.

(c) Compare the models for the two cities.

100. SALES A company that produces snowboards, which are seasonal products, forecasts monthly sales over the next 2 years to be $S = 23.1 + 0.442t + 4.3\cos(\pi t/6)$, where S is measured in thousands of units and t is the time in months, with $t = 1$ representing January 2010. Predict sales for each of the following months.

(a) February 2010 (b) February 2011

(c) June 2010 (d) June 2011

101. HARMONIC MOTION The displacement from equilibrium of an oscillating weight suspended by a spring and subject to the damping effect of friction is given by $y(t) = 2e^{-t}\cos 6t$, where y is the displacement (in centimeters) and t is the time (in seconds). Find the displacement when (a) $t = 0$, (b) $t = \frac{1}{4}$, and (c) $t = \frac{1}{2}$.

102. ELECTRIC CIRCUITS The current I (in amperes) when 100 volts is applied to a circuit is given by $I = 5e^{-2t}\sin t$, where t is the time (in seconds) after the voltage is applied. Approximate the current at $t = 0.7$ second after the voltage is applied.

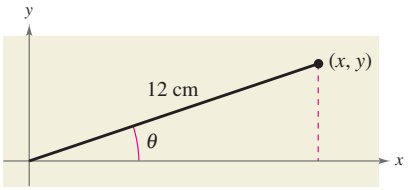
EXPLORATION

TRUE OR FALSE? In Exercises 103 and 104, determine whether the statement is true or false. Justify your answer.

103. In each of the four quadrants, the signs of the secant function and sine function will be the same.

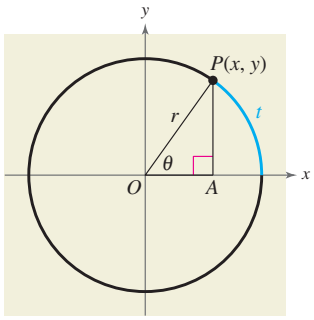
104. To find the reference angle for an angle θ (given in degrees), find the integer n such that $0 \le 360°n - \theta \le 360°$. The difference $360°n - \theta$ is the reference angle.

105. WRITING Consider an angle in standard position with $r = 12$ centimeters, as shown in the figure. Write a short paragraph describing the changes in the values of x, y, $\sin\theta$, $\cos\theta$, and $\tan\theta$ as θ increases continuously from 0° to 90°.



106. CAPSTONE Write a short paper in your own words explaining to a classmate how to evaluate the six trigonometric functions of any angle θ in standard position. Include an explanation of reference angles and how to use them, the signs of the functions in each of the four quadrants, and the trigonometric values of common angles. Be sure to include figures or diagrams in your paper.

107. THINK ABOUT IT The figure shows point $P(x, y)$ on a unit circle and right triangle OAP.



(a) Find $\sin t$ and $\cos t$ using the unit circle definitions of sine and cosine (from Section 4.2).

(b) What is the value of r? Explain.

(c) Use the definitions of sine and cosine given in this section to find $\sin\theta$ and $\cos\theta$. Write your answers in terms of x and y.

(d) Based on your answers to parts (a) and (c), what can you conclude?