4.3 RIGHT TRIANGLE TRIGONOMETRY

What You Should Learn

- Evaluate trigonometric functions of acute angles.
- Use fundamental trigonometric identities.
- Use a calculator to evaluate trigonometric functions.
- Use trigonometric functions to model and solve real-life problems.

The Six Trigonometric Functions

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0° < θ < 90° (θ lies in the first quadrant) and that for such angles the value of each trigonometric function is *positive*.

Right Triangle Definitions of Trigonometric Functions

Let θ be an *acute* angle of a right triangle. The six trigonometric functions of the angle θ are defined as follows. (Note that the functions in the second row are the *reciprocals* of the corresponding functions in the first row.)

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

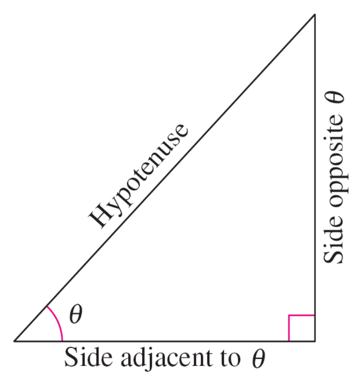
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

The abbreviations opp, adj, and hyp represent the lengths of the three sides of a right triangle.

opp = the length of the side *opposite* θ

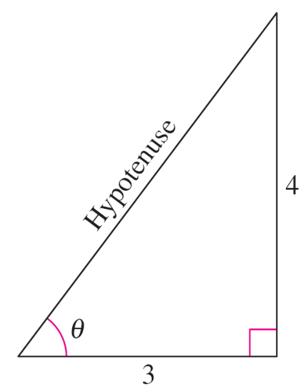
adj = the length of the side *adjacent to* θ

hyp = the length of the *hypotenuse*



Example 1 – *Evaluating Trigonometric Functions*

Use the triangle in the figure to find the values of the six trigonometric functions of θ.



Solution:

By the Pythagorean Theorem,

$(\text{hyp})^2 = (\text{opp})^2 + (\text{adj})^2$, it follows that

$$\begin{aligned} \text{hyp} &= \sqrt{4^2 + 3^2} \\ &= \sqrt{25} \\ &= 5. \end{aligned}$$

Example 1 – *Solution*

cont'd

So, the six trigonometric functions of θ are

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}.$$

The Six Trigonometric Functions

Sines, Cosines, and Tangents of Special Angles

$$\sin 30^\circ = \sin\frac{\pi}{6} = \frac{1}{2} \qquad \cos 30^\circ = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\sin 45^\circ = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \cos 45^\circ = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \tan 45^\circ = \tan\frac{\pi}{4} = 1$$

$$\sin 60^\circ = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \cos\frac{\pi}{3} = \frac{1}{2} \qquad \tan 60^\circ = \tan\frac{\pi}{3} = \sqrt{3}$$

If θ is an acute angle, *cofunctions of complementary angles are equal*.

$$\sin(90^\circ - \theta) = \cos\theta \qquad \cos(90^\circ - \theta) = \sin\theta$$

$$\tan(90^\circ - \theta) = \cot\theta \qquad \cot(90^\circ - \theta) = \tan\theta$$

$$\sec(90^\circ - \theta) = \csc\theta \qquad \csc(90^\circ - \theta) = \sec\theta$$

Trigonometric Identities

Trigonometric Identities

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Example 4 – *Applying Trigonometric Identities*

Let θ be an acute angle such that $\sin \theta = 0.6$. Find the values of (a) $\cos \theta$ and (b) $\tan \theta$ using trigonometric identities.

Solution:

a. To find the value of $\cos \theta$, use the Pythagorean identity

$$\sin^2 \theta + \cos^2 \theta = 1.$$

$$(0.6)^2 + \cos^2 \theta = 1$$ Substitute 0.6 for $\sin \theta$.

$$\cos^2 \theta = 1 - (0.6)^2$$ Subtract $(0.6)^2$ from each side.

$$= 0.64$$

Example 4 – *Solution*

cont'd

$$\cos\theta = \sqrt{0.64}$$

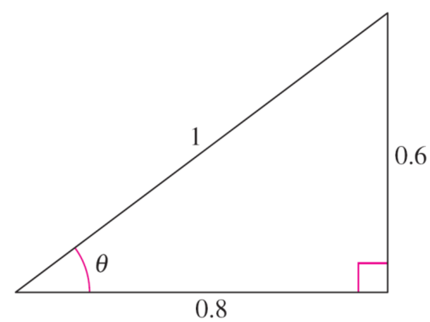
$$= 0.8.$$

Extract the positive square root.

b. $$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0.6}{0.8}$$

$$= 0.75.$$

Use the definitions of cos θ and tan θ, and the triangle shown in figure, to check these results.



Example 6 –

Evaluate sec(5° 40′ 12″).

Solution:

Begin by converting to decimal degree form.

[Recall that $1' = \frac{1}{60}(1°)$ and $1'' = \frac{1}{3600}(1°)$].

$$5° \; 40' \; 12'' = 5° + \left(\frac{40}{60}\right)^{\circ} + \left(\frac{12}{3600}\right)^{\circ}$$

$$= 5.67°$$

$$\sec(5° \; 40' \; 12'') = \sec 5.67°$$

Applications Involving Right Triangles

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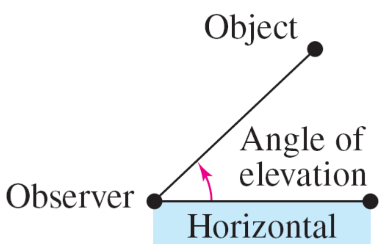
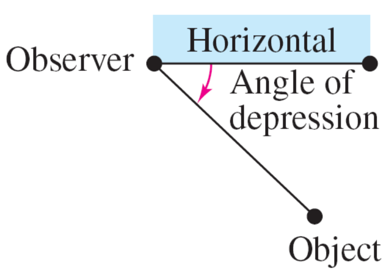


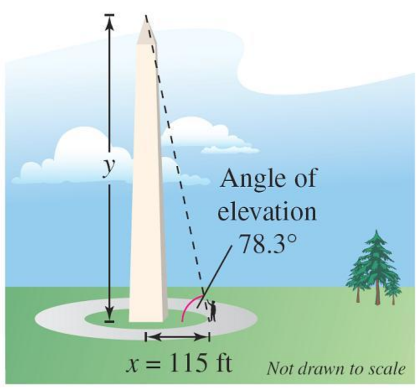
The **angle of elevation,** which represents the angle from the horizontal upward to an object.



For objects that lie below the horizontal, it is common to use the term **angle of depression**

Example 7 – *Using Trigonometry to Solve a Right Triangle*

A surveyor is standing 115 feet from the base of the Washington Monument, as shown in Figure 4.33. The surveyor measures the angle of elevation to the top of the monument as 78.3°. How tall is the Washington Monument?



Example 7 – *Solution*

$$\tan 78.3^\circ = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

where $x = 115$ and y is the height of the monument. So, the height of the Washington Monument is

$$y = x \tan 78.3^\circ$$

$$\approx 115(4.82882)$$

$$\approx 555 \text{ feet.}$$