4.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

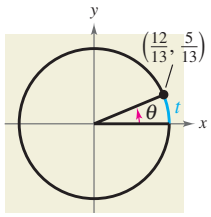
VOCABULARY: Fill in the blanks.

1. Each real number t corresponds to a point (x, y) on the _____ _____.
2. A function f is _____ if there exists a positive real number c such that $f(t + c) = f(t)$ for all t in the domain of f.
3. The smallest number c for which a function f is periodic is called the _____ of f.
4. A function f is _____ if $f(-t) = -f(t)$ and _____ if $f(-t) = f(t)$.

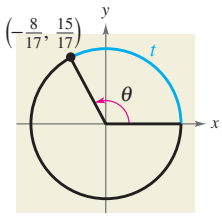
SKILLS AND APPLICATIONS

In Exercises 5–8, determine the exact values of the six trigonometric functions of the real number t.

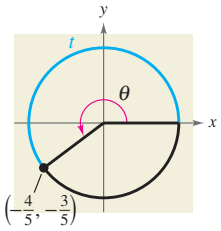
5.



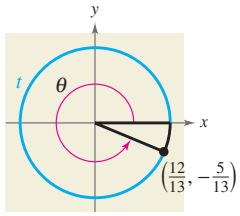
6.



7.



8.



In Exercises 9–16, find the point (x, y) on the unit circle that corresponds to the real number t.

9. $t = \frac{\pi}{2}$
10. $t = \pi$
11. $t = \frac{\pi}{4}$
12. $t = \frac{\pi}{3}$
13. $t = \frac{5\pi}{6}$
14. $t = \frac{3\pi}{4}$
15. $t = \frac{4\pi}{3}$
16. $t = \frac{5\pi}{3}$

In Exercises 17–26, evaluate (if possible) the sine, cosine, and tangent of the real number.

17. $t = \frac{\pi}{4}$
18. $t = \frac{\pi}{3}$
19. $t = -\frac{\pi}{6}$
20. $t = -\frac{\pi}{4}$
21. $t = -\frac{7\pi}{4}$
22. $t = -\frac{4\pi}{3}$
23. $t = \frac{11\pi}{6}$
24. $t = \frac{5\pi}{3}$
25. $t = -\frac{3\pi}{2}$
26. $t = -2\pi$

In Exercises 27–34, evaluate (if possible) the six trigonometric functions of the real number.

27. $t = \frac{2\pi}{3}$
28. $t = \frac{5\pi}{6}$
29. $t = \frac{4\pi}{3}$
30. $t = \frac{7\pi}{4}$
31. $t = \frac{3\pi}{4}$
32. $t = \frac{3\pi}{2}$
33. $t = -\frac{\pi}{2}$
34. $t = -\pi$

In Exercises 35–42, evaluate the trigonometric function using its period as an aid.

35. $\sin 4\pi$
36. $\cos 3\pi$
37. $\cos \frac{7\pi}{3}$
38. $\sin \frac{9\pi}{4}$
39. $\cos \frac{17\pi}{4}$
40. $\sin \frac{19\pi}{6}$
41. $\sin\left(-\frac{8\pi}{3}\right)$
42. $\cos\left(-\frac{9\pi}{4}\right)$

In Exercises 43–48, use the value of the trigonometric function to evaluate the indicated functions.

43. $\sin t = \frac{1}{2}$
    (a) $\sin(-t)$
    (b) $\csc(-t)$
44. $\sin(-t) = \frac{3}{8}$
    (a) $\sin t$
    (b) $\csc t$
45. $\cos(-t) = -\frac{1}{5}$
    (a) $\cos t$
    (b) $\sec(-t)$
46. $\cos t = -\frac{3}{4}$
    (a) $\cos(-t)$
    (b) $\sec(-t)$
47. $\sin t = \frac{4}{5}$
    (a) $\sin(\pi - t)$
    (b) $\sin(t + \pi)$
48. $\cos t = \frac{4}{5}$
    (a) $\cos(\pi - t)$
    (b) $\cos(t + \pi)$

In Exercises 49–58, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is set in the correct angle mode.)

49. $\sin\frac{\pi}{4}$

50. $\tan\frac{\pi}{3}$

51. $\cot\frac{\pi}{4}$

52. $\csc\frac{2\pi}{3}$

53. $\cos(-1.7)$

54. $\cos(-2.5)$

55. $\csc 0.8$

56. $\sec 1.8$

57. $\sec(-22.8)$

58. $\cot(-0.9)$

59. HARMONIC MOTION The displacement from equilibrium of an oscillating weight suspended by a spring is given by $y(t) = \frac{1}{4}\cos 6t$, where y is the displacement (in feet) and t is the time (in seconds). Find the displacements when (a) $t = 0$, (b) $t = \frac{1}{4}$, and (c) $t = \frac{1}{2}$.

60. HARMONIC MOTION The displacement from equilibrium of an oscillating weight suspended by a spring and subject to the damping effect of friction is given by $y(t) = \frac{1}{4}e^{-t}\cos 6t$, where y is the displacement (in feet) and t is the time (in seconds).

(a) Complete the table.

t	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
y					

(b) Use the *table* feature of a graphing utility to approximate the time when the weight reaches equilibrium.

(c) What appears to happen to the displacement as t increases?

EXPLORATION

TRUE OR FALSE? **In Exercises 61–64, determine whether the statement is true or false. Justify your answer.**

61. Because $\sin(-t) = -\sin t$, it can be said that the sine of a negative angle is a negative number.

62. $\tan a = \tan(a - 6\pi)$

63. The real number 0 corresponds to the point (0, 1) on the unit circle.

64. $\cos\left(-\frac{7\pi}{2}\right) = \cos\left(\pi + \frac{\pi}{2}\right)$

65. Let (x_1, y_1) and (x_2, y_2) be points on the unit circle corresponding to $t = t_1$ and $t = \pi - t_1$, respectively.

(a) Identify the symmetry of the points (x_1, y_1) and (x_2, y_2).

(b) Make a conjecture about any relationship between $\sin t_1$ and $\sin(\pi - t_1)$.

(c) Make a conjecture about any relationship between $\cos t_1$ and $\cos(\pi - t_1)$.

66. Use the unit circle to verify that the cosine and secant functions are even and that the sine, cosecant, tangent, and cotangent functions are odd.

67. Verify that $\cos 2t \neq 2\cos t$ by approximating $\cos 1.5$ and $2\cos 0.75$.

68. Verify that $\sin(t_1 + t_2) \neq \sin t_1 + \sin t_2$ by approximating $\sin 0.25$, $\sin 0.75$, and $\sin 1$.

69. THINK ABOUT IT Because $f(t) = \sin t$ is an odd function and $g(t) = \cos t$ is an even function, what can be said about the function $h(t) = f(t)g(t)$?

70. THINK ABOUT IT Because $f(t) = \sin t$ and $g(t) = \tan t$ are odd functions, what can be said about the function $h(t) = f(t)g(t)$?

71. GRAPHICAL ANALYSIS With your graphing utility in *radian* and *parametric* modes, enter the equations

$$X_{1T} = \cos T \quad \text{and} \quad Y_{1T} = \sin T$$

and use the following settings.

$$\text{Tmin} = 0,\ \text{Tmax} = 6.3,\ \text{Tstep} = 0.1$$
$$\text{Xmin} = -1.5,\ \text{Xmax} = 1.5,\ \text{Xscl} = 1$$
$$\text{Ymin} = -1,\ \text{Ymax} = 1,\ \text{Yscl} = 1$$

(a) Graph the entered equations and describe the graph.

(b) Use the *trace* feature to move the cursor around the graph. What do the t-values represent? What do the x- and y-values represent?

(c) What are the least and greatest values of x and y?

72. CAPSTONE A student you are tutoring has used a unit circle divided into 8 equal parts to complete the table for selected values of t. What is wrong?

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
x	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1
y	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0
$\sin t$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1
$\cos t$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0
$\tan t$	Undef.	1	0	-1	Undef.