4.1 RADIAN AND DEGREE MEASURE

What You Should Learn

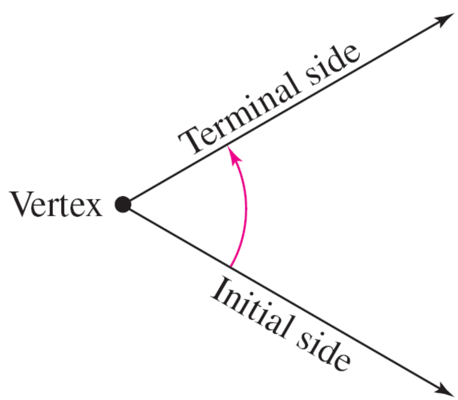
- Describe angles.
- Use radian measure.
- Use degree measure.
- Use angles to model and solve real-life problems.

Angles

Angles

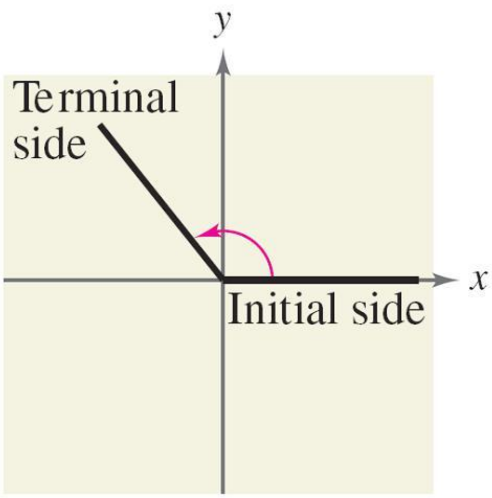
Trigonometry means "measurement of triangles."

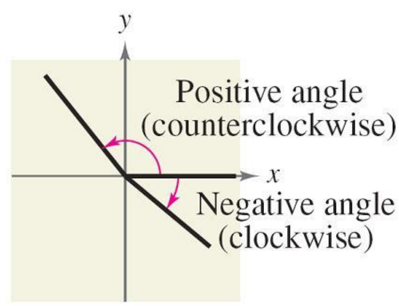
An **angle** is determined by rotating a ray (half-line) about its endpoint.

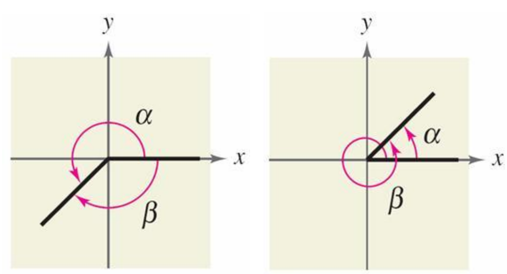


Angles

standard position







Same initial and terminal sides.
Such angles are **coterminal.**

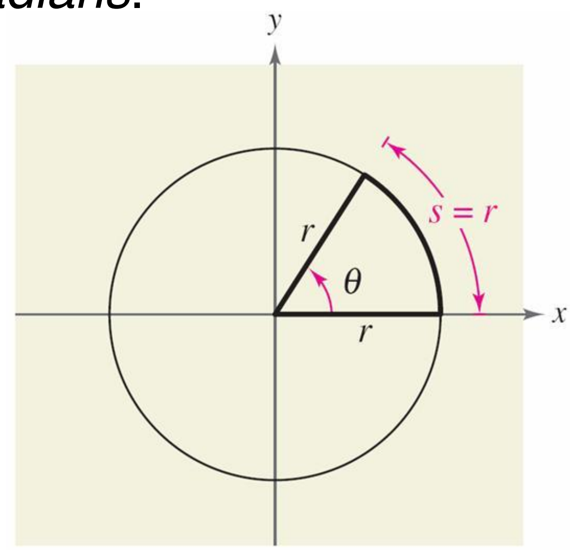
Radian Measure

Radian Measure

The **measure of an angle** is determined by the amount of rotation from the initial side to the terminal side.

One way to measure angles is in *radians*.

To define a radian, you can use a **central angle** of a circle, one whose vertex is the center of the circle.



Arc length = radius when θ = 1 radian

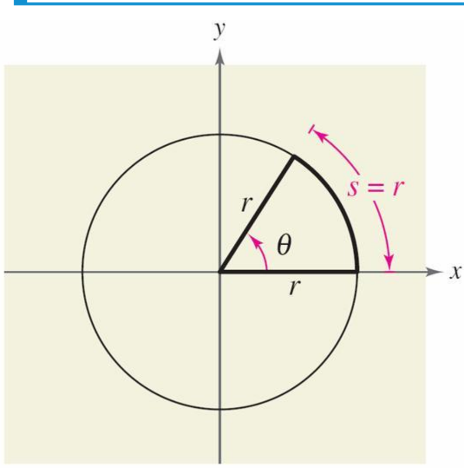
Radian Measure

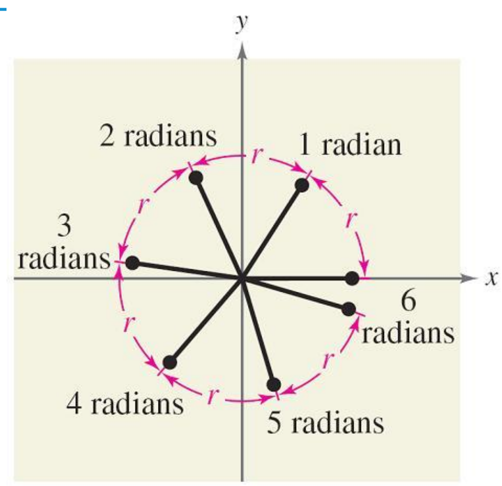
Definition of Radian

One **radian** is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle. See Figure 4.5. Algebraically, this means that

$$\theta = \frac{s}{r}$$

where θ is measured in radians.



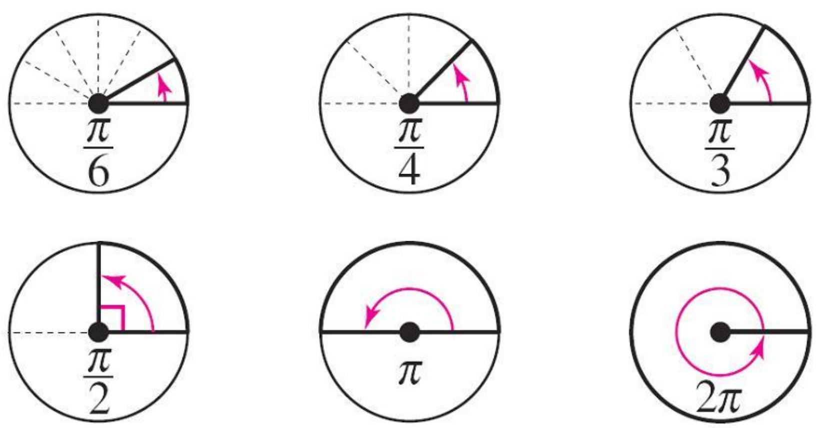


Radian Measure

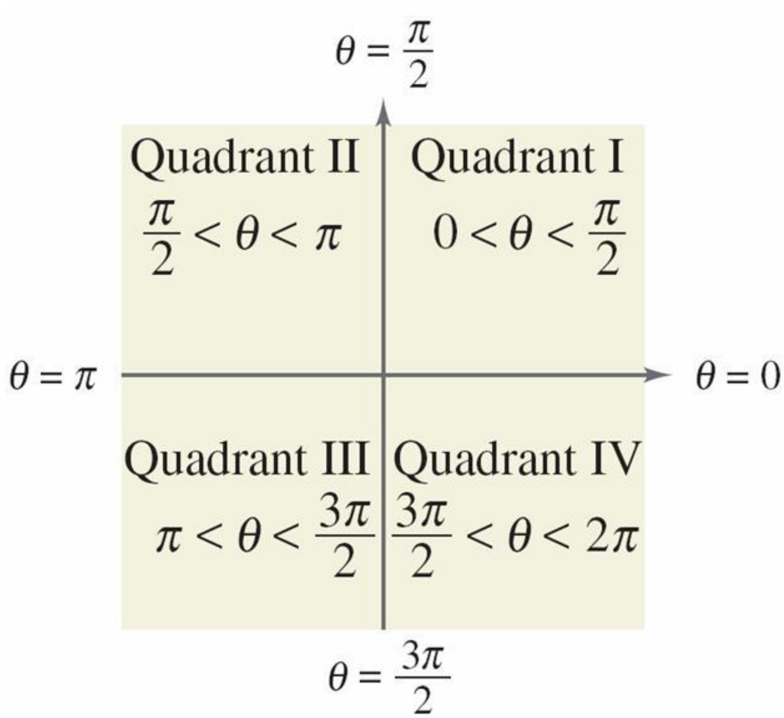
$$\frac{1}{2}\text{ revolution } = \frac{2\pi}{2} = \pi \text{ radians}$$

$$\frac{1}{4}\text{ revolution } = \frac{2\pi}{4} = \frac{\pi}{2} \text{ radians}$$

$$\frac{1}{6}\text{ revolution } = \frac{2\pi}{6} = \frac{\pi}{3} \text{ radians}$$



Radian Measure



Radian Measure

A given angle θ has infinitely many coterminal angles. For instance, $\theta = \pi/6$ is coterminal with

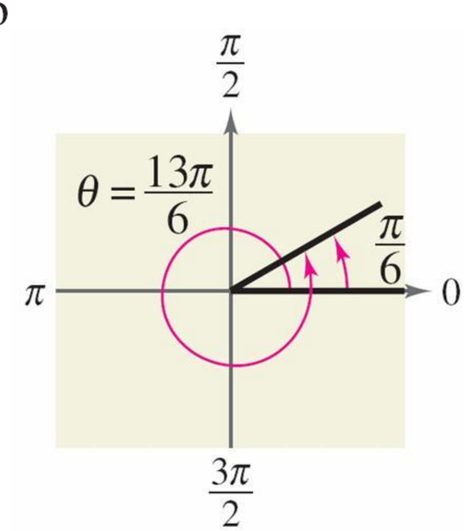
$$\frac{\pi}{6} + 2n\pi$$

where *n* is an integer.

Example 1 – *Sketching and Finding Coterminal Angles*

a. For the positive angle $13\pi/6$, subtract 2π to obtain a coterminal angle

$$\frac{13\pi}{6} - 2\pi = \frac{\pi}{6}.$$



Example 1 – *Sketching and Finding Coterminal Angles*

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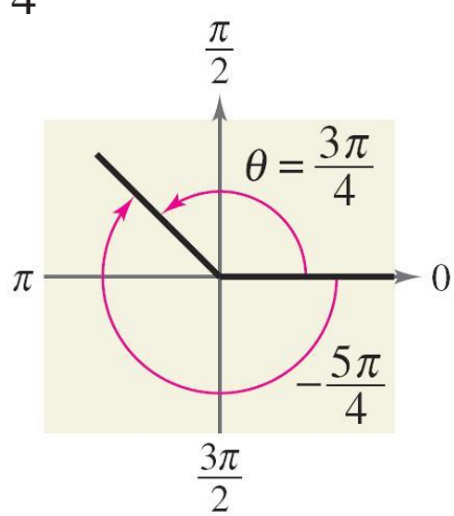
b. For the positive angle $3\pi/4$, subtract 2π to obtain a coterminal angle

$$\frac{3\pi}{4} - 2\pi = -\frac{5\pi}{4}.$$



Example 1 – *Sketching and Finding Coterminal Angles*
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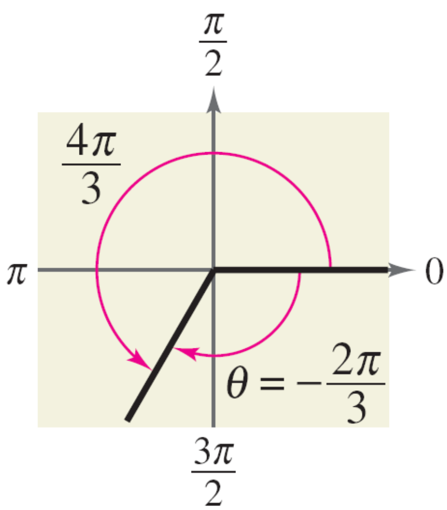
c. For the negative angle $-2\pi/3$, add 2π to obtain a coterminal angle

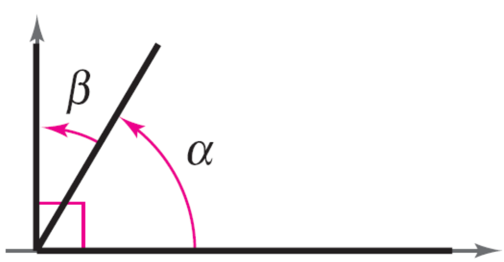
$$-\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}.$$



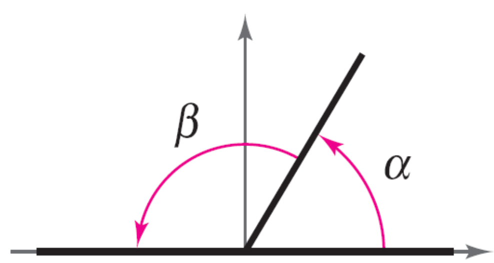
Radian Measure

Two positive angles α and β are **complementary** (complements of each other) if their sum is $\pi/2$.

Two positive angles are **supplementary** (supplements of each other) if their sum is π.



Complementary angles



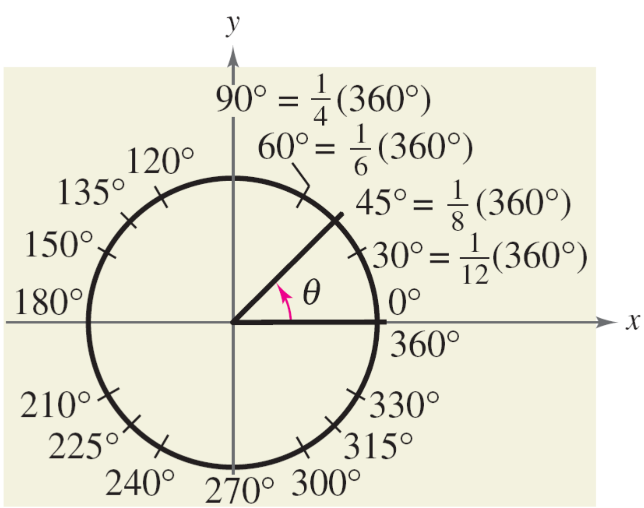
Supplementary angles

Degree Measure

Degree Measure

A second way to measure angles is in terms of **degrees,** denoted by the symbol °.

A measure of one degree (1°) is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution about the vertex.



Degree Measure

$$1^\circ = \frac{\pi}{180} \text{ rad} \qquad\qquad 1 \text{ rad} = \left(\frac{180^\circ}{\pi}\right)$$

When no units of angle measure are specified, *radian measure is implied*.

Degree Measure

Conversions Between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\dfrac{\pi \text{ rad}}{180°}$.

2. To convert radians to degrees, multiply radians by $\dfrac{180°}{\pi \text{ rad}}$.

To apply these two conversion rules, use the basic relationship π rad $= 180°$. (See Figure 4.14.)

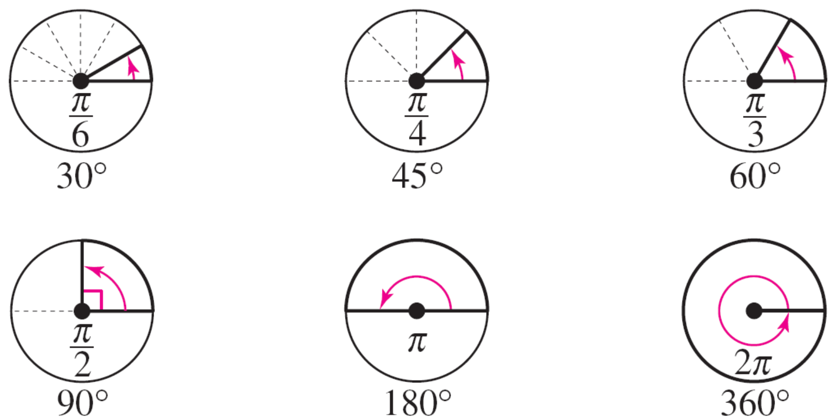


Figure 4.14

Example 3 – *Converting from Degrees to Radians*

a. $135° = (135 \text{ deg})\left(\dfrac{\pi \text{ rad}}{180 \text{ deg}}\right)$ Multiply by $\pi/180$.

$= \dfrac{3\pi}{4}$ radians

b. $540° = (540 \text{ deg})\left(\dfrac{\pi \text{ rad}}{180 \text{ deg}}\right)$ Multiply by $\pi/180$.

$= 3\pi$ radians

c. $-270° = (-270 \text{ deg})\left(\dfrac{\pi \text{ rad}}{180 \text{ deg}}\right)$ Multiply by $\pi/180$.

$= -\dfrac{3\pi}{2}$ radians

Applications

Applications

Arc Length

For a circle of radius r, a central angle θ intercepts an arc of length s given by

$$s = r\theta \qquad \text{Length of circular arc}$$

where θ is measured in radians. Note that if $r = 1$, then $s = \theta$, and the radian measure of θ equals the arc length.

Example 5 – *Finding Arc Length*

A circle has a radius of 4 inches. Find the length of the arc intercepted by a central angle of 240°, as shown in Figure 4.15.

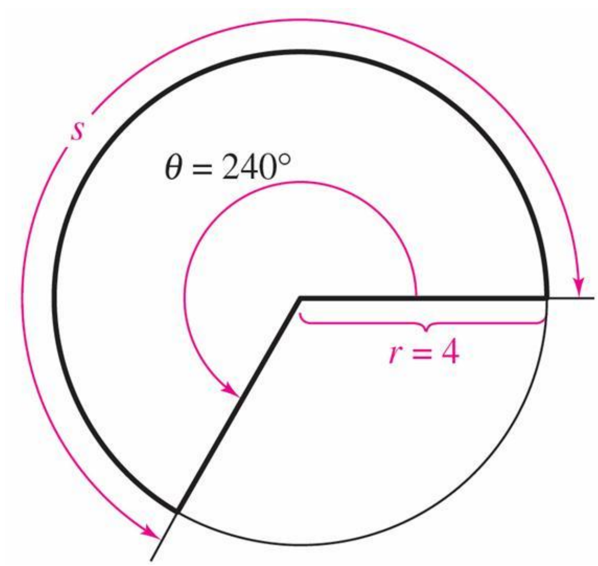


Figure 4.15

Example 5 – *Solution*

Convert 240° to radian measure:

$$240^\circ = (240 \text{ deg})\left(\frac{\pi \text{ rad}}{180 \text{ deg}}\right)$$

$$= \frac{4\pi}{3} \text{ radians}$$

Find the arc length to be

$s = r\theta$

$$= 4\left(\frac{4\pi}{3}\right)$$

$$= \frac{16\pi}{3} \approx 16.76 \text{ inches.}$$

Applications

Linear and Angular Speeds

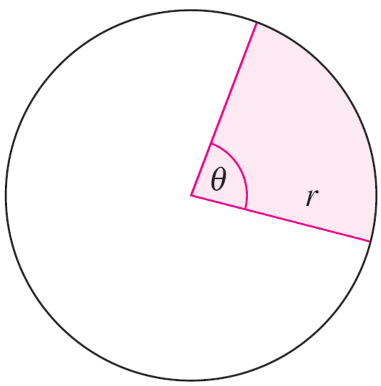
Consider a particle moving at a constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the **linear speed** v of the particle is

$$\text{Linear speed } v = \frac{\text{arc length}}{\text{time}} = \frac{s}{t}.$$

Moreover, if θ is the angle (in radian measure) corresponding to the arc length s, then the **angular speed** ω (the lowercase Greek letter omega) of the particle is

$$\text{Angular speed } \omega = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}.$$

Applications



A **sector** of a circle is the region bounded by two radii of the circle and their intercepted arc

Area of a Sector of a Circle

For a circle of radius r, the area A of a sector of the circle with central angle θ is given by

$$A = \frac{1}{2}r^2\theta$$

where θ is measured in radians.