5.5 EXERCISES

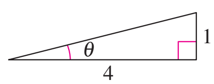
See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blank to complete the trigonometric formula.

1. $\sin 2u =$ ________
2. $\dfrac{1 + \cos 2u}{2} =$ ________
3. $\cos 2u =$ ________
4. $\dfrac{1 - \cos 2u}{1 + \cos 2u} =$ ________
5. $\sin \dfrac{u}{2} =$ ________
6. $\tan \dfrac{u}{2} =$ ________
7. $\cos u \cos v =$ ________
8. $\sin u \cos v =$ ________
9. $\sin u + \sin v =$ ________
10. $\cos u - \cos v =$ ________

SKILLS AND APPLICATIONS

In Exercises 11–18, use the figure to find the exact value of the trigonometric function.



11. $\cos 2\theta$
12. $\sin 2\theta$
13. $\tan 2\theta$
14. $\sec 2\theta$
15. $\csc 2\theta$
16. $\cot 2\theta$
17. $\sin 4\theta$
18. $\tan 4\theta$

In Exercises 19–28, find the exact solutions of the equation in the interval $[0, 2\pi)$.

19. $\sin 2x - \sin x = 0$
20. $\sin 2x + \cos x = 0$
21. $4 \sin x \cos x = 1$
22. $\sin 2x \sin x = \cos x$
23. $\cos 2x - \cos x = 0$
24. $\cos 2x + \sin x = 0$
25. $\sin 4x = -2 \sin 2x$
26. $(\sin 2x + \cos 2x)^2 = 1$
27. $\tan 2x - \cot x = 0$
28. $\tan 2x - 2 \cos x = 0$

In Exercises 29–36, use a double-angle formula to rewrite the expression.

29. $6 \sin x \cos x$
30. $\sin x \cos x$
31. $6 \cos^2 x - 3$
32. $\cos^2 x - \frac{1}{2}$
33. $4 - 8 \sin^2 x$
34. $10 \sin^2 x - 5$
35. $(\cos x + \sin x)(\cos x - \sin x)$
36. $(\sin x - \cos x)(\sin x + \cos x)$

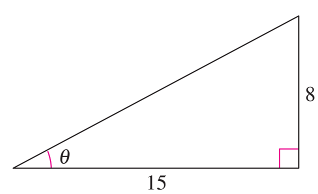
In Exercises 37–42, find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

37. $\sin u = -\dfrac{3}{5}, \quad \dfrac{3\pi}{2} < u < 2\pi$
38. $\cos u = -\dfrac{4}{5}, \quad \dfrac{\pi}{2} < u < \pi$
39. $\tan u = \dfrac{3}{5}, \quad 0 < u < \dfrac{\pi}{2}$
40. $\cot u = \sqrt{2}, \quad \pi < u < \dfrac{3\pi}{2}$
41. $\sec u = -2, \quad \dfrac{\pi}{2} < u < \pi$
42. $\csc u = 3, \quad \dfrac{\pi}{2} < u < \pi$

In Exercises 43–52, use the power-reducing formulas to rewrite the expression in terms of the first power of the cosine.

43. $\cos^4 x$
44. $\sin^4 2x$
45. $\cos^4 2x$
46. $\sin^8 x$
47. $\tan^4 2x$
48. $\sin^2 x \cos^4 x$
49. $\sin^2 2x \cos^2 2x$
50. $\tan^2 2x \cos^4 2x$
51. $\sin^4 x \cos^2 x$
52. $\sin^4 x \cos^4 x$

In Exercises 53–58, use the figure to find the exact value of the trigonometric function.



53. $\cos \dfrac{\theta}{2}$
54. $\sin \dfrac{\theta}{2}$
55. $\tan \dfrac{\theta}{2}$
56. $\sec \dfrac{\theta}{2}$
57. $\csc \dfrac{\theta}{2}$
58. $\cot \dfrac{\theta}{2}$

In Exercises 59–66, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

59. $75°$ **60.** $165°$

61. $112° 30'$ **62.** $67° 30'$

63. $\pi/8$ **64.** $\pi/12$

65. $3\pi/8$ **66.** $7\pi/12$

In Exercises 67–72, (a) determine the quadrant in which $u/2$ lies, and (b) find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ using the half-angle formulas.

67. $\cos u = \frac{7}{25}, \quad 0 < u < \frac{\pi}{2}$

68. $\sin u = \frac{5}{13}, \quad \frac{\pi}{2} < u < \pi$

69. $\tan u = -\frac{5}{12}, \quad \frac{3\pi}{2} < u < 2\pi$

70. $\cot u = 3, \quad \pi < u < \frac{3\pi}{2}$

71. $\csc u = -\frac{5}{3}, \quad \pi < u < \frac{3\pi}{2}$

72. $\sec u = \frac{7}{2}, \quad \frac{3\pi}{2} < u < 2\pi$

In Exercises 73–76, use the half-angle formulas to simplify the expression.

73. $\sqrt{\frac{1 - \cos 6x}{2}}$ **74.** $\sqrt{\frac{1 + \cos 4x}{2}}$

75. $-\sqrt{\frac{1 - \cos 8x}{1 + \cos 8x}}$ **76.** $-\sqrt{\frac{1 - \cos(x - 1)}{2}}$

In Exercises 77–80, find all solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to graph the equation and verify the solutions.

77. $\sin \frac{x}{2} + \cos x = 0$ **78.** $\sin \frac{x}{2} + \cos x - 1 = 0$

79. $\cos \frac{x}{2} - \sin x = 0$ **80.** $\tan \frac{x}{2} - \sin x = 0$

In Exercises 81–90, use the product-to-sum formulas to write the product as a sum or difference.

81. $\sin \frac{\pi}{3} \cos \frac{\pi}{6}$ **82.** $4 \cos \frac{\pi}{3} \sin \frac{5\pi}{6}$

83. $10 \cos 75° \cos 15°$ **84.** $6 \sin 45° \cos 15°$

85. $\sin 5\theta \sin 3\theta$ **86.** $3 \sin(-4\alpha) \sin 6\alpha$

87. $7 \cos(-5\beta) \sin 3\beta$ **88.** $\cos 2\theta \cos 4\theta$

89. $\sin(x + y) \sin(x - y)$ **90.** $\sin(x + y) \cos(x - y)$

In Exercises 91–98, use the sum-to-product formulas to write the sum or difference as a product.

91. $\sin 3\theta + \sin \theta$ **92.** $\sin 5\theta - \sin 3\theta$

93. $\cos 6x + \cos 2x$ **94.** $\cos x + \cos 4x$

95. $\sin(\alpha + \beta) - \sin(\alpha - \beta)$ **96.** $\cos(\phi + 2\pi) + \cos \phi$

97. $\cos\left(\theta + \frac{\pi}{2}\right) - \cos\left(\theta - \frac{\pi}{2}\right)$

98. $\sin\left(x + \frac{\pi}{2}\right) + \sin\left(x - \frac{\pi}{2}\right)$

In Exercises 99–102, use the sum-to-product formulas to find the exact value of the expression.

99. $\sin 75° + \sin 15°$ **100.** $\cos 120° + \cos 60°$

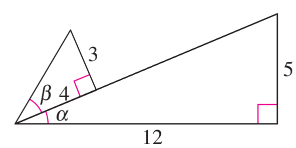
101. $\cos \frac{3\pi}{4} - \cos \frac{\pi}{4}$ **102.** $\sin \frac{5\pi}{4} - \sin \frac{3\pi}{4}$

In Exercises 103–106, find all solutions of the equation in the interval $[0, 2\pi)$. Use a graphing utility to graph the equation and verify the solutions.

103. $\sin 6x + \sin 2x = 0$ **104.** $\cos 2x - \cos 6x = 0$

105. $\frac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$ **106.** $\sin^2 3x - \sin^2 x = 0$

In Exercises 107–110, use the figure to find the exact value of the trigonometric function.



107. $\sin 2\alpha$ **108.** $\cos 2\beta$

109. $\cos(\beta/2)$ **110.** $\sin(\alpha + \beta)$

In Exercises 111–124, verify the identity.

111. $\csc 2\theta = \frac{\csc \theta}{2 \cos \theta}$ **112.** $\sec 2\theta = \frac{\sec^2 \theta}{2 - \sec^2 \theta}$

113. $\sin \frac{\alpha}{3} \cos \frac{\alpha}{3} = \frac{1}{2} \sin \frac{2\alpha}{3}$ **114.** $\frac{\cos 3\beta}{\cos \beta} = 1 - 4 \sin^2 \beta$

115. $1 + \cos 10y = 2 \cos^2 5y$

116. $\cos^4 x - \sin^4 x = \cos 2x$

117. $\cos 4\alpha = \cos^2 2\alpha - \sin^2 2\alpha$

118. $(\sin x + \cos x)^2 = 1 + \sin 2x$

119. $\tan \frac{u}{2} = \csc u - \cot u$

120. $\sec \frac{u}{2} = \pm\sqrt{\frac{2 \tan u}{\tan u + \sin u}}$

121. $\dfrac{\cos 4x + \cos 2x}{\sin 4x + \sin 2x} = \cot 3x$

122. $\dfrac{\sin x \pm \sin y}{\cos x + \cos y} = \tan \dfrac{x \pm y}{2}$

123. $\sin\left(\dfrac{\pi}{6} + x\right) + \sin\left(\dfrac{\pi}{6} - x\right) = \cos x$

124. $\cos\left(\dfrac{\pi}{3} + x\right) + \cos\left(\dfrac{\pi}{3} - x\right) = \cos x$

In Exercises 125–128, use a graphing utility to verify the identity. Confirm that it is an identity algebraically.

125. $\cos 3\beta = \cos^3 \beta - 3 \sin^2 \beta \cos \beta$

126. $\sin 4\beta = 4 \sin \beta \cos \beta(1 - 2 \sin^2 \beta)$

127. $(\cos 4x - \cos 2x)/(2 \sin 3x) = -\sin x$

128. $(\cos 3x - \cos x)/(\sin 3x - \sin x) = -\tan 2x$

In Exercises 129 and 130, graph the function by hand in the interval $[0, 2\pi]$ by using the power-reducing formulas.

129. $f(x) = \sin^2 x$

130. $f(x) = \cos^2 x$

In Exercises 131–134, write the trigonometric expression as an algebraic expression.

131. $\sin(2 \arcsin x)$

132. $\cos(2 \arccos x)$

133. $\cos(2 \arcsin x)$

134. $\sin(2 \arccos x)$

135. PROJECTILE MOTION The range of a projectile fired at an angle θ with the horizontal and with an initial velocity of v_0 feet per second is

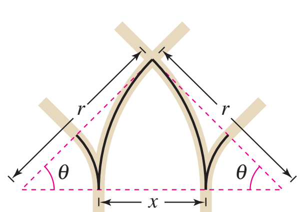
$$r = \frac{1}{32} v_0^2 \sin 2\theta$$

where r is measured in feet. An athlete throws a javelin at 75 feet per second. At what angle must the athlete throw the javelin so that the javelin travels 130 feet?

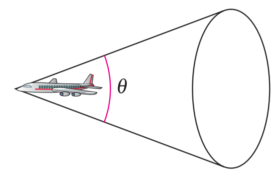
136. RAILROAD TRACK When two railroad tracks merge, the overlapping portions of the tracks are in the shapes of circular arcs (see figure). The radius of each arc r (in feet) and the angle θ are related by

$$\frac{x}{2} = 2r \sin^2 \frac{\theta}{2}.$$

Write a formula for x in terms of $\cos \theta$.



137. MACH NUMBER The mach number M of an airplane is the ratio of its speed to the speed of sound. When an airplane travels faster than the speed of sound, the sound waves form a cone behind the airplane (see figure). The mach number is related to the apex angle θ of the cone by $\sin(\theta/2) = 1/M$.



(a) Find the angle θ that corresponds to a mach number of 1.

(b) Find the angle θ that corresponds to a mach number of 4.5.

(c) The speed of sound is about 760 miles per hour. Determine the speed of an object with the mach numbers from parts (a) and (b).

(d) Rewrite the equation in terms of θ.

EXPLORATION

138. CAPSTONE Consider the function given by $f(x) = \sin^4 x + \cos^4 x$.

(a) Use the power-reducing formulas to write the function in terms of cosine to the first power.

(b) Determine another way of rewriting the function. Use a graphing utility to rule out incorrectly rewritten functions.

(c) Add a trigonometric term to the function so that it becomes a perfect square trinomial. Rewrite the function as a perfect square trinomial minus the term that you added. Use a graphing utility to rule out incorrectly rewritten functions.

(d) Rewrite the result of part (c) in terms of the sine of a double angle. Use a graphing utility to rule out incorrectly rewritten functions.

(e) When you rewrite a trigonometric expression, the result may not be the same as a friend's. Does this mean that one of you is wrong? Explain.

TRUE OR FALSE? **In Exercises 139 and 140, determine whether the statement is true or false. Justify your answer.**

139. Because the sine function is an odd function, for a negative number u, $\sin 2u = -2 \sin u \cos u$.

140. $\sin \dfrac{u}{2} = -\sqrt{\dfrac{1 - \cos u}{2}}$ when u is in the second quadrant.