5.4 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blank.

1. $\sin(u - v) =$ ________

2. $\cos(u + v) =$ ________

3. $\tan(u + v) =$ ________

4. $\sin(u + v) =$ ________

5. $\cos(u - v) =$ ________

6. $\tan(u - v) =$ ________

SKILLS AND APPLICATIONS

In Exercises 7–12, find the exact value of each expression.

7. (a) $\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$ (b) $\cos\frac{\pi}{4} + \cos\frac{\pi}{3}$

8. (a) $\sin\left(\frac{3\pi}{4} + \frac{5\pi}{6}\right)$ (b) $\sin\frac{3\pi}{4} + \sin\frac{5\pi}{6}$

9. (a) $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right)$ (b) $\sin\frac{7\pi}{6} - \sin\frac{\pi}{3}$

10. (a) $\cos(120° + 45°)$ (b) $\cos 120° + \cos 45°$

11. (a) $\sin(135° - 30°)$ (b) $\sin 135° - \cos 30°$

12. (a) $\sin(315° - 60°)$ (b) $\sin 315° - \sin 60°$

In Exercises 13–28, find the exact values of the sine, cosine, and tangent of the angle.

13. $\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$

14. $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$

15. $\frac{17\pi}{12} = \frac{9\pi}{4} - \frac{5\pi}{6}$

16. $-\frac{\pi}{12} = \frac{\pi}{6} - \frac{\pi}{4}$

17. $105° = 60° + 45°$

18. $165° = 135° + 30°$

19. $195° = 225° - 30°$

20. $255° = 300° - 45°$

21. $\frac{13\pi}{12}$

22. $-\frac{7\pi}{12}$

23. $-\frac{13\pi}{12}$

24. $\frac{5\pi}{12}$

25. $285°$

26. $-105°$

27. $-165°$

28. $15°$

In Exercises 29–36, write the expression as the sine, cosine, or tangent of an angle.

29. $\sin 3 \cos 1.2 - \cos 3 \sin 1.2$

30. $\cos\frac{\pi}{7}\cos\frac{\pi}{5} - \sin\frac{\pi}{7}\sin\frac{\pi}{5}$

31. $\sin 60° \cos 15° + \cos 60° \sin 15°$

32. $\cos 130° \cos 40° - \sin 130° \sin 40°$

33. $\dfrac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

34. $\dfrac{\tan 140° - \tan 60°}{1 + \tan 140° \tan 60°}$

35. $\dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

36. $\cos 3x \cos 2y + \sin 3x \sin 2y$

In Exercises 37–42, find the exact value of the expression.

37. $\sin\frac{\pi}{12}\cos\frac{\pi}{4} + \cos\frac{\pi}{12}\sin\frac{\pi}{4}$

38. $\cos\frac{\pi}{16}\cos\frac{3\pi}{16} - \sin\frac{\pi}{16}\sin\frac{3\pi}{16}$

39. $\sin 120° \cos 60° - \cos 120° \sin 60°$

40. $\cos 120° \cos 30° + \sin 120° \sin 30°$

41. $\dfrac{\tan(5\pi/6) - \tan(\pi/6)}{1 + \tan(5\pi/6)\tan(\pi/6)}$

42. $\dfrac{\tan 25° + \tan 110°}{1 - \tan 25° \tan 110°}$

In Exercises 43–50, find the exact value of the trigonometric function given that $\sin u = \frac{5}{13}$ and $\cos v = -\frac{3}{5}$. (Both u and v are in Quadrant II.)

43. $\sin(u + v)$

44. $\cos(u - v)$

45. $\cos(u + v)$

46. $\sin(v - u)$

47. $\tan(u + v)$

48. $\csc(u - v)$

49. $\sec(v - u)$

50. $\cot(u + v)$

In Exercises 51–56, find the exact value of the trigonometric function given that $\sin u = -\frac{7}{25}$ and $\cos v = -\frac{4}{5}$. (Both u and v are in Quadrant III.)

51. $\cos(u + v)$

52. $\sin(u + v)$

53. $\tan(u - v)$

54. $\cot(v - u)$

55. $\csc(u - v)$

56. $\sec(v - u)$

In Exercises 57–60, write the trigonometric expression as an algebraic expression.

57. $\sin(\arcsin x + \arccos x)$

58. $\sin(\arctan 2x - \arccos x)$

59. $\cos(\arccos x + \arcsin x)$

60. $\cos(\arccos x - \arctan x)$

In Exercises 61–70, prove the identity.

61. $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ **62.** $\sin\left(\frac{\pi}{2} + x\right) = \cos x$

63. $\sin\left(\frac{\pi}{6} + x\right) = \frac{1}{2}\left(\cos x + \sqrt{3}\sin x\right)$

64. $\cos\left(\frac{5\pi}{4} - x\right) = -\frac{\sqrt{2}}{2}(\cos x + \sin x)$

65. $\cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = 0$

66. $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\theta}{1 + \tan\theta}$

67. $\cos(x + y)\cos(x - y) = \cos^2 x - \sin^2 y$

68. $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$

69. $\sin(x + y) + \sin(x - y) = 2\sin x\cos y$

70. $\cos(x + y) + \cos(x - y) = 2\cos x\cos y$

In Exercises 71–74, simplify the expression algebraically and use a graphing utility to confirm your answer graphically.

71. $\cos\left(\frac{3\pi}{2} - x\right)$ **72.** $\cos(\pi + x)$

73. $\sin\left(\frac{3\pi}{2} + \theta\right)$ **74.** $\tan(\pi + \theta)$

In Exercises 75–84, find all solutions of the equation in the interval $[0, 2\pi)$.

75. $\sin(x + \pi) - \sin x + 1 = 0$

76. $\sin(x + \pi) - \sin x - 1 = 0$

77. $\cos(x + \pi) - \cos x - 1 = 0$

78. $\cos(x + \pi) - \cos x + 1 = 0$

79. $\sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$

80. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$

81. $\cos\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 1$

82. $\tan(x + \pi) + 2\sin(x + \pi) = 0$

83. $\sin\left(x + \frac{\pi}{2}\right) - \cos^2 x = 0$

84. $\cos\left(x - \frac{\pi}{2}\right) + \sin^2 x = 0$

In Exercises 85–88, use a graphing utility to approximate the solutions in the interval $[0, 2\pi)$.

85. $\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = 1$

86. $\tan(x + \pi) - \cos\left(x + \frac{\pi}{2}\right) = 0$

87. $\sin\left(x + \frac{\pi}{2}\right) + \cos^2 x = 0$

88. $\cos\left(x - \frac{\pi}{2}\right) - \sin^2 x = 0$

89. HARMONIC MOTION A weight is attached to a spring suspended vertically from a ceiling. When a driving force is applied to the system, the weight moves vertically from its equilibrium position, and this motion is modeled by

$$y = \frac{1}{3}\sin 2t + \frac{1}{4}\cos 2t$$

where y is the distance from equilibrium (in feet) and t is the time (in seconds).

(a) Use the identity

$$a\sin B\theta + b\cos B\theta = \sqrt{a^2 + b^2}\sin(B\theta + C)$$

where $C = \arctan(b/a)$, $a > 0$, to write the model in the form $y = \sqrt{a^2 + b^2}\sin(Bt + C)$.

(b) Find the amplitude of the oscillations of the weight.

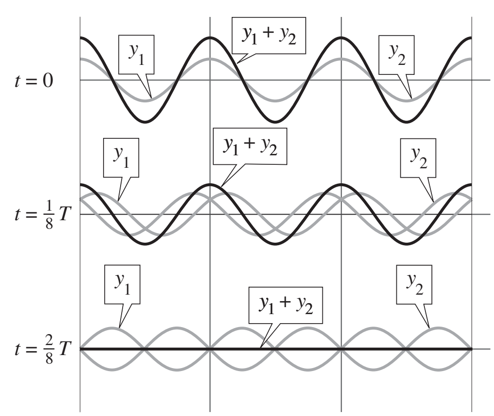
(c) Find the frequency of the oscillations of the weight.

90. STANDING WAVES The equation of a standing wave is obtained by adding the displacements of two waves traveling in opposite directions (see figure). Assume that each of the waves has amplitude A, period T, and wavelength λ. If the models for these waves are

$$y_1 = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{and} \quad y_2 = A\cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

show that

$$y_1 + y_2 = 2A\cos\frac{2\pi t}{T}\cos\frac{2\pi x}{\lambda}.$$



EXPLORATION

TRUE OR FALSE? In Exercises 91–94, determine whether the statement is true or false. Justify your answer.

91. $\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

92. $\cos(u \pm v) = \cos u \cos v \pm \sin u \sin v$

93. $\tan\left(x - \dfrac{\pi}{4}\right) = \dfrac{\tan x + 1}{1 - \tan x}$

94. $\sin\left(x - \dfrac{\pi}{2}\right) = -\cos x$

In Exercises 95–98, verify the identity.

95. $\cos(n\pi + \theta) = (-1)^n \cos \theta$, n is an integer

96. $\sin(n\pi + \theta) = (-1)^n \sin \theta$, n is an integer

97. $a \sin B\theta + b \cos B\theta = \sqrt{a^2 + b^2} \sin(B\theta + C)$, where $C = \arctan(b/a)$ and $a > 0$

98. $a \sin B\theta + b \cos B\theta = \sqrt{a^2 + b^2} \cos(B\theta - C)$, where $C = \arctan(a/b)$ and $b > 0$

In Exercises 99–102, use the formulas given in Exercises 97 and 98 to write the trigonometric expression in the following forms.

(a) $\sqrt{a^2 + b^2} \sin(B\theta + C)$ (b) $\sqrt{a^2 + b^2} \cos(B\theta - C)$

99. $\sin \theta + \cos \theta$

100. $3 \sin 2\theta + 4 \cos 2\theta$

101. $12 \sin 3\theta + 5 \cos 3\theta$

102. $\sin 2\theta + \cos 2\theta$

In Exercises 103 and 104, use the formulas given in Exercises 97 and 98 to write the trigonometric expression in the form $a \sin B\theta + b \cos B\theta$.

103. $2 \sin\left(\theta + \dfrac{\pi}{4}\right)$

104. $5 \cos\left(\theta - \dfrac{\pi}{4}\right)$

105. Verify the following identity used in calculus.

$$\frac{\cos(x + h) - \cos x}{h} = \frac{\cos x(\cos h - 1)}{h} - \frac{\sin x \sin h}{h}$$

106. Let $x = \pi/6$ in the identity in Exercise 105 and define the functions f and g as follows.

$$f(h) = \frac{\cos[(\pi/6) + h] - \cos(\pi/6)}{h}$$

$$g(h) = \cos \frac{\pi}{6}\left(\frac{\cos h - 1}{h}\right) - \sin \frac{\pi}{6}\left(\frac{\sin h}{h}\right)$$

(a) What are the domains of the functions f and g?

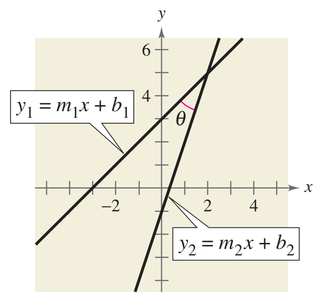
(b) Use a graphing utility to complete the table.

h	0.5	0.2	0.1	0.05	0.02	0.01
$f(h)$						
$g(h)$						

(c) Use a graphing utility to graph the functions f and g.

(d) Use the table and the graphs to make a conjecture about the values of the functions f and g as $h \to 0$.

In Exercises 107 and 108, use the figure, which shows two lines whose equations are $y_1 = m_1x + b_1$ and $y_2 = m_2x + b_2$. Assume that both lines have positive slopes. Derive a formula for the angle between the two lines. Then use your formula to find the angle between the given pair of lines.



107. $y = x$ and $y = \sqrt{3}x$

108. $y = x$ and $y = \dfrac{1}{\sqrt{3}}x$

In Exercises 109 and 110, use a graphing utility to graph y_1 and y_2 in the same viewing window. Use the graphs to determine whether $y_1 = y_2$. Explain your reasoning.

109. $y_1 = \cos(x + 2)$, $y_2 = \cos x + \cos 2$

110. $y_1 = \sin(x + 4)$, $y_2 = \sin x + \sin 4$

111. PROOF

(a) Write a proof of the formula for $\sin(u + v)$.

(b) Write a proof of the formula for $\sin(u - v)$.

112. CAPSTONE Give an example to justify each statement.

(a) $\sin(u + v) \neq \sin u + \sin v$

(b) $\sin(u - v) \neq \sin u - \sin v$

(c) $\cos(u + v) \neq \cos u + \cos v$

(d) $\cos(u - v) \neq \cos u - \cos v$

(e) $\tan(u + v) \neq \tan u + \tan v$

(f) $\tan(u - v) \neq \tan u - \tan v$