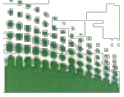




5.4

SUM AND DIFFERENCE FORMULAS



Using Sum and Difference Formulas



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Sum and Difference Formulas

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \qquad \tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$



Example 1 – Evaluating a Trigonometric Function

Find the exact value of $\sin \frac{\pi}{12}$.

Solution:

To find the *exact* value of $\sin \frac{\pi}{12}$, use the fact that

$$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}.$$

Consequently, the formula for $\sin(u - v)$ yields

$$\sin \frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$



Example 1 – Solution

cont'd

$$\begin{aligned} &= \sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4} \\ &= \frac{\sqrt{3}}{2} \left(\frac{\sqrt{2}}{2} \right) - \frac{1}{2} \left(\frac{\sqrt{2}}{2} \right) \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}. \end{aligned}$$

Try checking this result on your calculator. You will find that $\sin \frac{\pi}{12} \approx 0.259$.



Using Sum and Difference Formulas

$$= \sin x.$$

$$\cos\left(\frac{\pi}{2} - x\right)$$



Example 5 – Proving a Cofunction identity

Prove the cofunction identity $\cos\left(\frac{\pi}{2} - x\right) = \sin x$.

Solution:

Using the formula for $\cos(u - v)$, you have

$$\begin{aligned}\cos\left(\frac{\pi}{2} - x\right) &= \cos \frac{\pi}{2} \cos x + \sin \frac{\pi}{2} \sin x \\ &= (0)(\cos x) + (1)(\sin x) \\ &= \sin x.\end{aligned}$$



Using Sum and Difference Formulas

Sum and difference formulas can be used to rewrite expressions such as

$$\sin\left(\theta + \frac{n\pi}{2}\right) \text{ and } \cos\left(\theta + \frac{n\pi}{2}\right), \quad \text{where } n \text{ is an integer}$$

as expressions involving only $\sin \theta$ or $\cos \theta$.

The resulting formulas are called **reduction formulas**.



Example 7 – Solving a Trigonometric Equation

Find all solutions of $\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = -1$

in the interval $[0, 2\pi)$.

Solution:

Using sum and difference formulas, rewrite the equation as

$$\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4} + \sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4} = -1$$

$$2 \sin x \cos \frac{\pi}{4} = -1$$



Example 7 – Solution

cont'd

$$2(\sin x)\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{2}$$

$$\sin x = -\frac{1}{\sqrt{2}}$$

$$\sin x = -\frac{\sqrt{2}}{2}.$$

So, the only solutions in the interval $[0, 2\pi)$ are

$$x = \frac{5\pi}{4} \quad \text{and} \quad x = \frac{7\pi}{4} .$$



More...

Find all solutions of the equations in the interval $[0, 2\pi)$

1. $\cos\left(x - \frac{\pi}{2}\right) + \sin^2 x = 0$

2. $\tan(x + \pi) + 2 \sin(x + \pi) = 0$

Write the trigonometric expression as an algebraic expression:

$\sin(\arctan 2x - \arccos x)$