5.4 SUM AND DIFFERENCE FORMULAS

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$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \qquad \tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Example 1 – *Evaluating a Trigonometric Function*

Find the exact value of $\sin \frac{\pi}{12}$.

Solution:

To find the *exact* value of $\sin \frac{\pi}{12}$, use the fact that

$$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}.$$

Consequently, the formula for sin(*u* – *v*) yields

$$\sin \frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

Example 1 – *Solution*

cont'd

$$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}.$$

Try checking this result on your calculator. You will find that $\sin\frac{\pi}{12} \approx 0.259$.

Using Sum and Difference Formulas

= sin *x*.

$$\cos\left(\frac{\pi}{2} - x\right)$$

Example 5 – *Proving a Cofunction identity*

Prove the cofunction identity $\cos\left(\frac{\pi}{2} - x\right)$ = sin *x*.

Solution:

Using the formula for cos(*u* – *v*), you have

$$\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$$

$$= (0)(\cos x) + (1)(\sin x)$$

$$= \sin x.$$

Using Sum and Difference Formulas

Sum and difference formulas can be used to rewrite expressions such as

$\sin\left(\theta + \frac{n\pi}{2}\right)$ and $\cos\left(\theta + \frac{n\pi}{2}\right)$, where n is an integer

as expressions involving only $\sin \theta$ or $\cos \theta$.

The resulting formulas are called **reduction formulas.**

Example 7 – *Solving a Trigonometric Equation*

Find all solutions of $\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = -1$

in the interval $[0, 2\pi)$.

Solution:

Using sum and difference formulas, rewrite the equation as

$$\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4} + \sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4} = -1$$

$$2 \sin x \cos \frac{\pi}{4} = -1$$

Example 7 – *Solution*

cont'd

$$2(\sin x)\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{2}$$

$$\sin x = -\frac{1}{\sqrt{2}}$$

$$\sin x = -\frac{\sqrt{2}}{2}.$$

So, the only solutions in the interval $[0, 2\pi)$ are

$x = \frac{5\pi}{4}$ and $x = \frac{7\pi}{4}$.

More…

Find all solutions of the equations in the interval $[0,2\pi)$

1. $\cos(x - \frac{\pi}{2}) + \sin^2 x = 0$
2. $\tan(x + \pi) + 2\sin(x + \pi) = 0$

Write the trigonometric expression as an algebraic expression:
$\sin(\arctan 2x - \arccos x)$