5.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY

In Exercises 1 and 2, fill in the blanks.

1. An equation that is true for all real values in its domain is called an ________.

2. An equation that is true for only some values in its domain is called a ________ ________.

In Exercises 3–8, fill in the blank to complete the trigonometric identity.

3. $\dfrac{1}{\cot u} =$ ________

4. $\dfrac{\cos u}{\sin u} =$ ________

5. $\sin^2 u +$ ________ $= 1$

6. $\cos\left(\dfrac{\pi}{2} - u\right) =$ ________

7. $\csc(-u) =$ ________

8. $\sec(-u) =$ ________

SKILLS AND APPLICATIONS

In Exercises 9–50, verify the identity.

9. $\tan t \cot t = 1$

10. $\sec y \cos y = 1$

11. $\cot^2 y(\sec^2 y - 1) = 1$

12. $\cos x + \sin x \tan x = \sec x$

13. $(1 + \sin \alpha)(1 - \sin \alpha) = \cos^2 \alpha$

14. $\cos^2 \beta - \sin^2 \beta = 2\cos^2 \beta - 1$

15. $\cos^2 \beta - \sin^2 \beta = 1 - 2\sin^2 \beta$

16. $\sin^2 \alpha - \sin^4 \alpha = \cos^2 \alpha - \cos^4 \alpha$

17. $\dfrac{\tan^2 \theta}{\sec \theta} = \sin \theta \tan \theta$

18. $\dfrac{\cot^3 t}{\csc t} = \cos t(\csc^2 t - 1)$

19. $\dfrac{\cot^2 t}{\csc t} = \dfrac{1 - \sin^2 t}{\sin t}$

20. $\dfrac{1}{\tan \beta} + \tan \beta = \dfrac{\sec^2 \beta}{\tan \beta}$

21. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \cos^3 x\sqrt{\sin x}$

22. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^5 x \tan^3 x$

23. $\dfrac{\cot x}{\sec x} = \csc x - \sin x$

24. $\dfrac{\sec \theta - 1}{1 - \cos \theta} = \sec \theta$

25. $\csc x - \sin x = \cos x \cot x$

26. $\sec x - \cos x = \sin x \tan x$

27. $\dfrac{1}{\tan x} + \dfrac{1}{\cot x} = \tan x + \cot x$

28. $\dfrac{1}{\sin x} - \dfrac{1}{\csc x} = \csc x - \sin x$

29. $\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta} = 2\sec \theta$

30. $\dfrac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \csc \theta$

31. $\dfrac{1}{\cos x + 1} + \dfrac{1}{\cos x - 1} = -2\csc x \cot x$

32. $\cos x - \dfrac{\cos x}{1 - \tan x} = \dfrac{\sin x \cos x}{\sin x - \cos x}$

33. $\tan\left(\dfrac{\pi}{2} - \theta\right)\tan \theta = 1$

34. $\dfrac{\cos[(\pi/2) - x]}{\sin[(\pi/2) - x]} = \tan x$

35. $\dfrac{\tan x \cot x}{\cos x} = \sec x$

36. $\dfrac{\csc(-x)}{\sec(-x)} = -\cot x$

37. $(1 + \sin y)[1 + \sin(-y)] = \cos^2 y$

38. $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\cot x + \cot y}{\cot x \cot y - 1}$

39. $\dfrac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$

40. $\dfrac{\cos x - \cos y}{\sin x + \sin y} + \dfrac{\sin x - \sin y}{\cos x + \cos y} = 0$

41. $\sqrt{\dfrac{1 + \sin \theta}{1 - \sin \theta}} = \dfrac{1 + \sin \theta}{|\cos \theta|}$

42. $\sqrt{\dfrac{1 - \cos \theta}{1 + \cos \theta}} = \dfrac{1 - \cos \theta}{|\sin \theta|}$

43. $\cos^2 \beta + \cos^2\left(\dfrac{\pi}{2} - \beta\right) = 1$

44. $\sec^2 y - \cot^2\left(\dfrac{\pi}{2} - y\right) = 1$

45. $\sin t \csc\left(\dfrac{\pi}{2} - t\right) = \tan t$

46. $\sec^2\left(\dfrac{\pi}{2} - x\right) - 1 = \cot^2 x$

47. $\tan(\sin^{-1} x) = \dfrac{x}{\sqrt{1 - x^2}}$

48. $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$

49. $\tan\left(\sin^{-1} \dfrac{x - 1}{4}\right) = \dfrac{x - 1}{\sqrt{16 - (x - 1)^2}}$

50. $\tan\left(\cos^{-1} \dfrac{x + 1}{2}\right) = \dfrac{\sqrt{4 - (x + 1)^2}}{x + 1}$

ERROR ANALYSIS In Exercises 51 and 52, describe the error(s).

51. $(1 + \tan x)[1 + \cot(-x)]$
$= (1 + \tan x)(1 + \cot x)$
$= 1 + \cot x + \tan x + \tan x \cot x$
$= 1 + \cot x + \tan x + 1$
$= 2 + \cot x + \tan x$

52. $\dfrac{1 + \sec(-\theta)}{\sin(-\theta) + \tan(-\theta)} = \dfrac{1 - \sec\theta}{\sin\theta - \tan\theta}$
$= \dfrac{1 - \sec\theta}{(\sin\theta)[1 - (1/\cos\theta)]}$
$= \dfrac{1 - \sec\theta}{\sin\theta(1 - \sec\theta)}$
$= \dfrac{1}{\sin\theta} = \csc\theta$

In Exercises 53–60, (a) use a graphing utility to graph each side of the equation to determine whether the equation is an identity, (b) use the *table* feature of a graphing utility to determine whether the equation is an identity, and (c) confirm the results of parts (a) and (b) algebraically.

53. $(1 + \cot^2 x)(\cos^2 x) = \cot^2 x$

54. $\csc x(\csc x - \sin x) + \dfrac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x$

55. $2 + \cos^2 x - 3\cos^4 x = \sin^2 x(3 + 2\cos^2 x)$

56. $\tan^4 x + \tan^2 x - 3 = \sec^2 x(4\tan^2 x - 3)$

57. $\csc^4 x - 2\csc^2 x + 1 = \cot^4 x$

58. $(\sin^4 \beta - 2\sin^2 \beta + 1)\cos\beta = \cos^5 \beta$

59. $\dfrac{1 + \cos x}{\sin x} = \dfrac{\sin x}{1 - \cos x}$ **60.** $\dfrac{\cot\alpha}{\csc\alpha + 1} = \dfrac{\csc\alpha + 1}{\cot\alpha}$

In Exercises 61–64, verify the identity.

61. $\tan^5 x = \tan^3 x \sec^2 x - \tan^3 x$

62. $\sec^4 x \tan^2 x = (\tan^2 x + \tan^4 x)\sec^2 x$

63. $\cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x)\cos x$

64. $\sin^4 x + \cos^4 x = 1 - 2\cos^2 x + 2\cos^4 x$

In Exercises 65–68, use the cofunction identities to evaluate the expression without using a calculator.

65. $\sin^2 25° + \sin^2 65°$ **66.** $\cos^2 55° + \cos^2 35°$

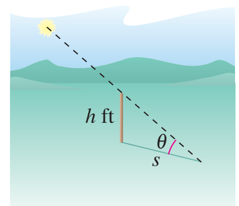
67. $\cos^2 20° + \cos^2 52° + \cos^2 38° + \cos^2 70°$

68. $\tan^2 63° + \cot^2 16° - \sec^2 74° - \csc^2 27°$

69. RATE OF CHANGE The rate of change of the function $f(x) = \sin x + \csc x$ with respect to change in the variable x is given by the expression $\cos x - \csc x \cot x$. Show that the expression for the rate of change can also be $-\cos x \cot^2 x$.

70. SHADOW LENGTH The length s of a shadow cast by a vertical gnomon (a device used to tell time) of height h when the angle of the sun above the horizon is θ (see figure) can be modeled by the equation

$$s = \frac{h\sin(90° - \theta)}{\sin\theta}.$$



(a) Verify that the equation for s is equal to $h \cot\theta$.

(b) Use a graphing utility to complete the table. Let $h = 5$ feet.

θ	15°	30°	45°	60°	75°	90°
s						

(c) Use your table from part (b) to determine the angles of the sun that result in the maximum and minimum lengths of the shadow.

(d) Based on your results from part (c), what time of day do you think it is when the angle of the sun above the horizon is 90°?

EXPLORATION

TRUE OR FALSE? In Exercises 71 and 72, determine whether the statement is true or false. Justify your answer.

71. There can be more than one way to verify a trigonometric identity.

72. The equation $\sin^2\theta + \cos^2\theta = 1 + \tan^2\theta$ is an identity because $\sin^2(0) + \cos^2(0) = 1$ and $1 + \tan^2(0) = 1$.

THINK ABOUT IT In Exercises 73–77, explain why the equation is not an identity and find one value of the variable for which the equation is not true.

73. $\sin\theta = \sqrt{1 - \cos^2\theta}$ **74.** $\tan\theta = \sqrt{\sec^2\theta - 1}$

75. $1 - \cos\theta = \sin\theta$ **76.** $\csc\theta - 1 = \cot\theta$

77. $1 + \tan\theta = \sec\theta$

78. CAPSTONE Write a short paper in your own words explaining to a classmate the difference between a trigonometric identity and a conditional equation. Include suggestions on how to verify a trigonometric identity.

5.3 SOLVING TRIGONOMETRIC EQUATIONS

What you should learn

- Use standard algebraic techniques to solve trigonometric equations.
- Solve trigonometric equations of quadratic type.
- Solve trigonometric equations involving multiple angles.
- Use inverse trigonometric functions to solve trigonometric equations.

Why you should learn it

You can use trigonometric equations to solve a variety of real-life problems. For instance, in Exercise 92 on page 396, you can solve a trigonometric equation to help answer questions about monthly sales of skiing equipment.

Tom Stillo/Index Stock Imagery/Photo Library

Introduction

To solve a trigonometric equation, use standard algebraic techniques such as collecting like terms and factoring. Your preliminary goal in solving a trigonometric equation is to *isolate* the trigonometric function in the equation. For example, to solve the equation $2 \sin x = 1$, divide each side by 2 to obtain

$$\sin x = \frac{1}{2}.$$

To solve for x, note in Figure 5.6 that the equation $\sin x = \frac{1}{2}$ has solutions $x = \pi/6$ and $x = 5\pi/6$ in the interval $[0, 2\pi)$. Moreover, because $\sin x$ has a period of 2π, there are infinitely many other solutions, which can be written as

$$x = \frac{\pi}{6} + 2n\pi \quad \text{and} \quad x = \frac{5\pi}{6} + 2n\pi \qquad \text{General solution}$$

where n is an integer, as shown in Figure 5.6.

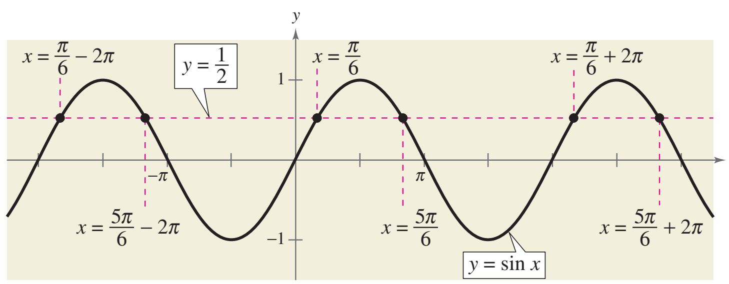


FIGURE 5.6

Another way to show that the equation $\sin x = \frac{1}{2}$ has infinitely many solutions is indicated in Figure 5.7. Any angles that are coterminal with $\pi/6$ or $5\pi/6$ will also be solutions of the equation.

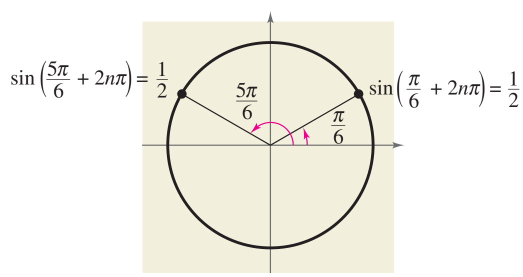


FIGURE 5.7

When solving trigonometric equations, you should write your answer(s) using exact values rather than decimal approximations.