5.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blank to complete the trigonometric identity.

1. $\dfrac{\sin u}{\cos u} = $ ________
2. $\dfrac{1}{\csc u} = $ ________
3. $\dfrac{1}{\tan u} = $ ________
4. $\dfrac{1}{\cos u} = $ ________
5. $1 + $ ________ $= \csc^2 u$
6. $1 + \tan^2 u = $ ________
7. $\sin\left(\dfrac{\pi}{2} - u\right) = $ ________
8. $\sec\left(\dfrac{\pi}{2} - u\right) = $ ________
9. $\cos(-u) = $ ________
10. $\tan(-u) = $ ________

SKILLS AND APPLICATIONS

In Exercises 11–24, use the given values to evaluate (if possible) all six trigonometric functions.

11. $\sin x = \dfrac{1}{2}, \quad \cos x = \dfrac{\sqrt{3}}{2}$
12. $\tan x = \dfrac{\sqrt{3}}{3}, \quad \cos x = -\dfrac{\sqrt{3}}{2}$
13. $\sec \theta = \sqrt{2}, \quad \sin \theta = -\dfrac{\sqrt{2}}{2}$
14. $\csc \theta = \frac{25}{7}, \quad \tan \theta = \frac{7}{24}$
15. $\tan x = \frac{8}{15}, \quad \sec x = -\frac{17}{15}$
16. $\cot \phi = -3, \quad \sin \phi = \dfrac{\sqrt{10}}{10}$
17. $\sec \phi = \dfrac{3}{2}, \quad \csc \phi = -\dfrac{3\sqrt{5}}{5}$
18. $\cos\left(\dfrac{\pi}{2} - x\right) = \dfrac{3}{5}, \quad \cos x = \dfrac{4}{5}$
19. $\sin(-x) = -\dfrac{1}{3}, \quad \tan x = -\dfrac{\sqrt{2}}{4}$
20. $\sec x = 4, \quad \sin x > 0$
21. $\tan \theta = 2, \quad \sin \theta < 0$
22. $\csc \theta = -5, \quad \cos \theta < 0$
23. $\sin \theta = -1, \quad \cot \theta = 0$
24. $\tan \theta$ is undefined, $\quad \sin \theta > 0$

In Exercises 25–30, match the trigonometric expression with one of the following.

(a) $\sec x$ (b) -1 (c) $\cot x$
(d) 1 (e) $-\tan x$ (f) $\sin x$

25. $\sec x \cos x$
26. $\tan x \csc x$
27. $\cot^2 x - \csc^2 x$
28. $(1 - \cos^2 x)(\csc x)$
29. $\dfrac{\sin(-x)}{\cos(-x)}$
30. $\dfrac{\sin[(\pi/2) - x]}{\cos[(\pi/2) - x]}$

In Exercises 31–36, match the trigonometric expression with one of the following.

(a) $\csc x$ (b) $\tan x$ (c) $\sin^2 x$
(d) $\sin x \tan x$ (e) $\sec^2 x$ (f) $\sec^2 x + \tan^2 x$

31. $\sin x \sec x$
32. $\cos^2 x(\sec^2 x - 1)$
33. $\sec^4 x - \tan^4 x$
34. $\cot x \sec x$
35. $\dfrac{\sec^2 x - 1}{\sin^2 x}$
36. $\dfrac{\cos^2[(\pi/2) - x]}{\cos x}$

In Exercises 37–58, use the fundamental identities to simplify the expression. There is more than one correct form of each answer.

37. $\cot \theta \sec \theta$
38. $\cos \beta \tan \beta$
39. $\tan(-x) \cos x$
40. $\sin x \cot(-x)$
41. $\sin \phi(\csc \phi - \sin \phi)$
42. $\sec^2 x(1 - \sin^2 x)$
43. $\dfrac{\cot x}{\csc x}$
44. $\dfrac{\csc \theta}{\sec \theta}$
45. $\dfrac{1 - \sin^2 x}{\csc^2 x - 1}$
46. $\dfrac{1}{\tan^2 x + 1}$
47. $\dfrac{\tan \theta \cot \theta}{\sec \theta}$
48. $\dfrac{\sin \theta \csc \theta}{\tan \theta}$
49. $\sec \alpha \cdot \dfrac{\sin \alpha}{\tan \alpha}$
50. $\dfrac{\tan^2 \theta}{\sec^2 \theta}$
51. $\cos\left(\dfrac{\pi}{2} - x\right) \sec x$
52. $\cot\left(\dfrac{\pi}{2} - x\right) \cos x$
53. $\dfrac{\cos^2 y}{1 - \sin y}$
54. $\cos t(1 + \tan^2 t)$
55. $\sin \beta \tan \beta + \cos \beta$
56. $\csc \phi \tan \phi + \sec \phi$
57. $\cot u \sin u + \tan u \cos u$
58. $\sin \theta \sec \theta + \cos \theta \csc \theta$

In Exercises 59–70, factor the expression and use the fundamental identities to simplify. There is more than one correct form of each answer.

59. $\tan^2 x - \tan^2 x \sin^2 x$

60. $\sin^2 x \csc^2 x - \sin^2 x$

61. $\sin^2 x \sec^2 x - \sin^2 x$

62. $\cos^2 x + \cos^2 x \tan^2 x$

63. $\dfrac{\sec^2 x - 1}{\sec x - 1}$

64. $\dfrac{\cos^2 x - 4}{\cos x - 2}$

65. $\tan^4 x + 2\tan^2 x + 1$

66. $1 - 2\cos^2 x + \cos^4 x$

67. $\sin^4 x - \cos^4 x$

68. $\sec^4 x - \tan^4 x$

69. $\csc^3 x - \csc^2 x - \csc x + 1$

70. $\sec^3 x - \sec^2 x - \sec x + 1$

In Exercises 71–74, perform the multiplication and use the fundamental identities to simplify. There is more than one correct form of each answer.

71. $(\sin x + \cos x)^2$

72. $(\cot x + \csc x)(\cot x - \csc x)$

73. $(2\csc x + 2)(2\csc x - 2)$

74. $(3 - 3\sin x)(3 + 3\sin x)$

In Exercises 75–80, perform the addition or subtraction and use the fundamental identities to simplify. There is more than one correct form of each answer.

75. $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x}$

76. $\dfrac{1}{\sec x + 1} - \dfrac{1}{\sec x - 1}$

77. $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x}$

78. $\dfrac{\tan x}{1 + \sec x} + \dfrac{1 + \sec x}{\tan x}$

79. $\tan x + \dfrac{\cos x}{1 + \sin x}$

80. $\tan x - \dfrac{\sec^2 x}{\tan x}$

In Exercises 81–84, rewrite the expression so that it is not in fractional form. There is more than one correct form of each answer.

81. $\dfrac{\sin^2 y}{1 - \cos y}$

82. $\dfrac{5}{\tan x + \sec x}$

83. $\dfrac{3}{\sec x - \tan x}$

84. $\dfrac{\tan^2 x}{\csc x + 1}$

NUMERICAL AND GRAPHICAL ANALYSIS In Exercises 85–88, use a graphing utility to complete the table and graph the functions. Make a conjecture about y_1 and y_2.

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1							
y_2							

85. $y_1 = \cos\left(\dfrac{\pi}{2} - x\right), \quad y_2 = \sin x$

86. $y_1 = \sec x - \cos x, \quad y_2 = \sin x \tan x$

87. $y_1 = \dfrac{\cos x}{1 - \sin x}, \quad y_2 = \dfrac{1 + \sin x}{\cos x}$

88. $y_1 = \sec^4 x - \sec^2 x, \quad y_2 = \tan^2 x + \tan^4 x$

In Exercises 89–92, use a graphing utility to determine which of the six trigonometric functions is equal to the expression. Verify your answer algebraically.

89. $\cos x \cot x + \sin x$

90. $\sec x \csc x - \tan x$

91. $\dfrac{1}{\sin x}\left(\dfrac{1}{\cos x} - \cos x\right)$

92. $\dfrac{1}{2}\left(\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta}\right)$

In Exercises 93–104, use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

93. $\sqrt{9 - x^2}, \quad x = 3\cos\theta$

94. $\sqrt{64 - 16x^2}, \quad x = 2\cos\theta$

95. $\sqrt{16 - x^2}, \quad x = 4\sin\theta$

96. $\sqrt{49 - x^2}, \quad x = 7\sin\theta$

97. $\sqrt{x^2 - 9}, \quad x = 3\sec\theta$

98. $\sqrt{x^2 - 4}, \quad x = 2\sec\theta$

99. $\sqrt{x^2 + 25}, \quad x = 5\tan\theta$

100. $\sqrt{x^2 + 100}, \quad x = 10\tan\theta$

101. $\sqrt{4x^2 + 9}, \quad 2x = 3\tan\theta$

102. $\sqrt{9x^2 + 25}, \quad 3x = 5\tan\theta$

103. $\sqrt{2 - x^2}, \quad x = \sqrt{2}\sin\theta$

104. $\sqrt{10 - x^2}, \quad x = \sqrt{10}\sin\theta$

In Exercises 105–108, use the trigonometric substitution to write the algebraic equation as a trigonometric equation of θ, where $-\pi/2 < \theta < \pi/2$. Then find $\sin\theta$ and $\cos\theta$.

105. $3 = \sqrt{9 - x^2}, \quad x = 3\sin\theta$

106. $3 = \sqrt{36 - x^2}, \quad x = 6\sin\theta$

107. $2\sqrt{2} = \sqrt{16 - 4x^2}, \quad x = 2\cos\theta$

108. $-5\sqrt{3} = \sqrt{100 - x^2}, \quad x = 10\cos\theta$

In Exercises 109–112, use a graphing utility to solve the equation for θ, where $0 \le \theta < 2\pi$.

109. $\sin\theta = \sqrt{1 - \cos^2\theta}$

110. $\cos\theta = -\sqrt{1 - \sin^2\theta}$

111. $\sec\theta = \sqrt{1 + \tan^2\theta}$

112. $\csc\theta = \sqrt{1 + \cot^2\theta}$

In Exercises 113–118, rewrite the expression as a single logarithm and simplify the result.

113. $\ln|\cos x| - \ln|\sin x|$ **114.** $\ln|\sec x| + \ln|\sin x|$

115. $\ln|\sin x| + \ln|\cot x|$ **116.** $\ln|\tan x| + \ln|\csc x|$

117. $\ln|\cot t| + \ln(1 + \tan^2 t)$

118. $\ln(\cos^2 t) + \ln(1 + \tan^2 t)$

In Exercises 119–122, use a calculator to demonstrate the identity for each value of θ.

119. $\csc^2 \theta - \cot^2 \theta = 1$

(a) $\theta = 132°$ (b) $\theta = \dfrac{2\pi}{7}$

120. $\tan^2 \theta + 1 = \sec^2 \theta$

(a) $\theta = 346°$ (b) $\theta = 3.1$

121. $\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin \theta$

(a) $\theta = 80°$ (b) $\theta = 0.8$

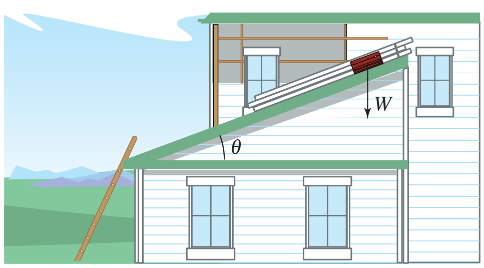
122. $\sin(-\theta) = -\sin \theta$

(a) $\theta = 250°$ (b) $\theta = \frac{1}{2}$

123. FRICTION The forces acting on an object weighing W units on an inclined plane positioned at an angle of θ with the horizontal (see figure) are modeled by

$$\mu W \cos \theta = W \sin \theta$$

where μ is the coefficient of friction. Solve the equation for μ and simplify the result.



124. RATE OF CHANGE The rate of change of the function $f(x) = -x + \tan x$ is given by the expression $-1 + \sec^2 x$. Show that this expression can also be written as $\tan^2 x$.

125. RATE OF CHANGE The rate of change of the function $f(x) = \sec x + \cos x$ is given by the expression $\sec x \tan x - \sin x$. Show that this expression can also be written as $\sin x \tan^2 x$.

126. RATE OF CHANGE The rate of change of the function $f(x) = -\csc x - \sin x$ is given by the expression $\csc x \cot x - \cos x$. Show that this expression can also be written as $\cos x \cot^2 x$.

EXPLORATION

TRUE OR FALSE? In Exercises 127 and 128, determine whether the statement is true or false. Justify your answer.

127. The even and odd trigonometric identities are helpful for determining whether the value of a trigonometric function is positive or negative.

128. A cofunction identity can be used to transform a tangent function so that it can be represented by a cosecant function.

In Exercises 129–132, fill in the blanks. (*Note:* The notation $x \to c^+$ indicates that x approaches c from the right and $x \to c^-$ indicates that x approaches c from the left.)

129. As $x \to \dfrac{\pi}{2}^-$, $\sin x \to$ ____ and $\csc x \to$ ____.

130. As $x \to 0^+$, $\cos x \to$ ____ and $\sec x \to$ ____.

131. As $x \to \dfrac{\pi}{2}^-$, $\tan x \to$ ____ and $\cot x \to$ ____.

132. As $x \to \pi^+$, $\sin x \to$ ____ and $\csc x \to$ ____.

In Exercises 133–138, determine whether or not the equation is an identity, and give a reason for your answer.

133. $\cos \theta = \sqrt{1 - \sin^2 \theta}$ **134.** $\cot \theta = \sqrt{\csc^2 \theta + 1}$

135. $\dfrac{(\sin k\theta)}{(\cos k\theta)} = \tan \theta$, k is a constant.

136. $\dfrac{1}{(5 \cos \theta)} = 5 \sec \theta$

137. $\sin \theta \csc \theta = 1$ **138.** $\csc^2 \theta = 1$

139. Use the trigonometric substitution $u = a \sin \theta$, where $-\pi/2 < \theta < \pi/2$ and $a > 0$, to simplify the expression $\sqrt{a^2 - u^2}$.

140. Use the trigonometric substitution $u = a \tan \theta$, where $-\pi/2 < \theta < \pi/2$ and $a > 0$, to simplify the expression $\sqrt{a^2 + u^2}$.

141. Use the trigonometric substitution $u = a \sec \theta$, where $0 < \theta < \pi/2$ and $a > 0$, to simplify the expression $\sqrt{u^2 - a^2}$.

142. CAPSTONE

(a) Use the definitions of sine and cosine to derive the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$.

(b) Use the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ to derive the other Pythagorean identities, $1 + \tan^2 \theta = \sec^2 \theta$ and $1 + \cot^2 \theta = \csc^2 \theta$. Discuss how to remember these identities and other fundamental identities.