


5.1

USING FUNDAMENTAL IDENTITIES



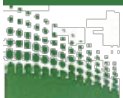
5.2

VERIFYING TRIGONOMETRIC IDENTITIES



What You Should Learn

- Recognize and write the fundamental trigonometric identities.
- Use the fundamental trigonometric identities to evaluate trigonometric functions, simplify trigonometric expressions, and rewrite trigonometric expressions.



Introduction

A graphic showing a stylized building or grid pattern in shades of green and white, positioned to the left of the title.

Introduction

We will learn how to use the fundamental identities to do the following.

1. Evaluate trigonometric functions.
2. Simplify trigonometric expressions.
3. Develop additional trigonometric identities.
4. Solve trigonometric equations.



Introduction

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u} \quad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u} \quad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \quad 1 + \tan^2 u = \sec^2 u \quad 1 + \cot^2 u = \csc^2 u$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u \qquad \tan(-u) = -\tan u$$

$$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u \qquad \cot(-u) = -\cot u$$



Introduction

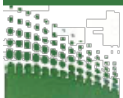
Pythagorean identities are sometimes used in radical form such as

$$\sin u = \pm \sqrt{1 - \cos^2 u}$$

or

$$\tan u = \pm \sqrt{\sec^2 u - 1}$$

where the sign depends on the choice of u .

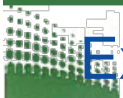


Using the Fundamental Identities



Using the Fundamental Identities

One common application of trigonometric identities is to use given values of trigonometric functions to evaluate other trigonometric functions.



Example 1 – Using Identities to Evaluate a Function

Use the values $\sec u = -\frac{3}{2}$ and $\tan u > 0$ to find the values of all six trigonometric functions.

Solution:

Using a reciprocal identity, you have

$$\cos u = \frac{1}{\sec u} = \frac{1}{-3/2} = -\frac{2}{3}.$$

Using a Pythagorean identity, you have

$$\sin^2 u = 1 - \cos^2 u \quad \text{Pythagorean identity}$$



Example 1 – Solution

cont'd

$$\begin{aligned} &= 1 - \left(-\frac{2}{3}\right)^2 && \text{Substitute } -\frac{2}{3} \text{ for } \cos u. \\ &= 1 - \frac{4}{9} && \text{Simplify.} \\ &= \frac{5}{9}. \end{aligned}$$

Because $\sec u < 0$ and $\tan u > 0$, it follows that u lies in Quadrant III.

Moreover, because $\sin u$ is negative when u is in Quadrant III, you can choose the negative root and obtain $\sin u = -\sqrt{5}/3$.



Example 1 – Solution

cont'd

Now, knowing the values of the sine and cosine, you can find the values of all six trigonometric functions.

$$\sin u = -\frac{\sqrt{5}}{3}$$

$$\cos u = -\frac{2}{3}$$

$$\tan u = \frac{\sin u}{\cos u} = \frac{-\sqrt{5}/3}{-2/3} = \frac{\sqrt{5}}{2}$$



Example 1 – Solution

cont'd

$$\csc u = \frac{1}{\sin u} = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$$

$$\sec u = \frac{1}{\cos u} = -\frac{3}{2}$$

$$\cot u = \frac{1}{\tan u} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$



Example 2 – Simplifying a Trigonometric Expression

Simplify $\sin x \cos^2 x - \sin x$.

Solution:

First factor out a common monomial factor and then use a fundamental identity.

$$\begin{aligned}\sin x \cos^2 x - \sin x &= \sin x (\cos^2 x - 1) && \text{Factor out common monomial factor.} \\ &= -\sin x (1 - \cos^2 x) && \text{Factor out } -1. \\ &= -\sin x (\sin^2 x) && \text{Pythagorean identity} \\ &= -\sin^3 x && \text{Multiply.}\end{aligned}$$



Example 7 – Rewriting a Trigonometric Expression

Rewrite $\frac{1}{1 + \sin x}$ so that it is *not* in fractional form.

Solution:

From the Pythagorean identity $\cos^2 x = 1 - \sin^2 x = (1 - \sin x)(1 + \sin x)$, you can see that multiplying both the numerator and the denominator by $(1 - \sin x)$ will produce a monomial denominator.

$$\frac{1}{1 + \sin x} = \frac{1}{1 + \sin x} \cdot \frac{1 - \sin x}{1 - \sin x}$$

Multiply numerator and denominator by $(1 - \sin x)$.

$$= \frac{1 - \sin x}{1 - \sin^2 x}$$

Multiply.



Example 7 – Solution

cont'd

$$= \frac{1 - \sin x}{\cos^2 x}$$

Pythagorean identity

$$= \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}$$

Write as separate fractions.

$$= \frac{1}{\cos^2 x} - \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

Product of fractions

$$= \sec^2 x - \tan x \sec x$$

Reciprocal and quotient identities



Verifying Trigonometric Identities

Guidelines for Verifying Trigonometric Identities

1. Work with one side of the equation at a time. It is often better to work with the more complicated side first.
2. Look for opportunities to factor an expression, add fractions, square a binomial, or create a monomial denominator.
3. Look for opportunities to use the fundamental identities. Note which functions are in the final expression you want. Sines and cosines pair up well, as do secants and tangents, and cosecants and cotangents.
4. If the preceding guidelines do not help, try converting all terms to sines and cosines.
5. Always try *something*. Even paths that lead to dead ends provide insights.



Example 1 – Verifying a Trigonometric Identity

Verify the identity $(\sec^2 \theta - 1)/(\sec^2 \theta) = \sin^2 \theta$.

Solution:

$$\frac{\sec^2 \theta - 1}{\sec^2 \theta} = \frac{(\tan^2 \theta + 1) - 1}{\sec^2 \theta} \quad \text{Pythagorean identity}$$

$$= \frac{\tan^2 \theta}{\sec^2 \theta} \quad \text{Simplify.}$$

$$= \tan^2 \theta (\cos^2 \theta) \quad \text{Reciprocal identity}$$

$$= \frac{\sin^2 \theta}{\cancel{(\cos^2 \theta)}} \cancel{(\cos^2 \theta)} \quad \text{Quotient identity}$$

$$= \sin^2 \theta$$