5.1 USING FUNDAMENTAL IDENTITIES

5.2 VERIFYING TRIGONOMETRIC IDENTITIES

What You Should Learn

- Recognize and write the fundamental trigonometric identities.
- Use the fundamental trigonometric identities to evaluate trigonometric functions, simplify trigonometric expressions, and rewrite trigonometric expressions.

Introduction

Introduction

We will learn how to use the fundamental identities to do the following.

1. Evaluate trigonometric functions.

2. Simplify trigonometric expressions.

3. Develop additional trigonometric identities.

4. Solve trigonometric equations.

Introduction

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \qquad 1 + \tan^2 u = \sec^2 u \qquad 1 + \cot^2 u = \csc^2 u$$

Introduction

cont'd

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u \qquad \tan(-u) = -\tan u$$

$$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u \qquad \cot(-u) = -\cot u$$

Introduction

Pythagorean identities are sometimes used in radical form such as

$$\sin u = \pm\sqrt{1 - \cos^2 u}$$

or

$$\tan u = \pm\sqrt{\sec^2 u - 1}$$

where the sign depends on the choice of u.

Using the Fundamental Identities

Using the Fundamental Identities

One common application of trigonometric identities is to use given values of trigonometric functions to evaluate other trigonometric functions.

Example 1 – *Using Identities to Evaluate a Function*

Use the values $\sec u = -\frac{3}{2}$ and tan $u > 0$ to find the values of all six trigonometric functions.

Solution:

Using a reciprocal identity, you have

$$\cos u = \frac{1}{\sec u} = \frac{1}{-3/2} = -\frac{2}{3}.$$

Using a Pythagorean identity, you have

$$\sin^2 u = 1 - \cos^2 u$$ Pythagorean identity

Example 1 – *Solution*

cont'd

$$= 1 - \left(-\frac{2}{3}\right)^2 \qquad \text{Substitute } -\tfrac{2}{3} \text{ for } \cos u.$$

$$= 1 - \frac{4}{9} \qquad \text{Simplify.}$$

$$= \frac{5}{9}.$$

Because sec $u < 0$ and tan $u > 0$, it follows that u lies in Quadrant III.

Moreover, because sin u is negative when u is in Quadrant III, you can choose the negative root and obtain $\sin u = -\sqrt{5}/3$.

Example 1 – *Solution*

cont'd

Now, knowing the values of the sine and cosine, you can find the values of all six trigonometric functions.

$$\sin u = -\frac{\sqrt{5}}{3}$$

$$\cos u = -\frac{2}{3}$$

$$\tan u = \frac{\sin u}{\cos u} = \frac{-\sqrt{5}/3}{-2/3} = \frac{\sqrt{5}}{2}$$

Example 1 – *Solution*

cont'd

$$\csc u = \frac{1}{\sin u} = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$$

$$\sec u = \frac{1}{\cos u} = -\frac{3}{2}$$

$$\cot u = \frac{1}{\tan u} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

Example 2 – *Simplifying a Trigonometric Expression*

Simplify $\sin x \cos^2 x - \sin x$.

Solution:

First factor out a common monomial factor and then use a fundamental identity.

$\sin x \cos^2 x - \sin x = \sin x\,(\cos^2 x - 1)$ — Factor out common monomial factor.

$= -\sin x(1 - \cos^2 x)$ — Factor out –1.

$= -\sin x(\sin^2 x)$ — Pythagorean identity

$= -\sin^3 x$ — Multiply.

Example 7 – *Rewriting a Trigonometric Expression*

Rewrite $\dfrac{1}{1 + \sin x}$ so that it is *not* in fractional form.

Solution:

From the Pythagorean identity $\cos^2 x = 1 - \sin^2 x = (1 - \sin x)(1 + \sin x)$, you can see that multiplying both the numerator and the denominator by $(1 - \sin x)$ will produce a monomial denominator.

$$\frac{1}{1 + \sin x} = \frac{1}{1 + \sin x} \cdot \frac{1 - \sin x}{1 - \sin x}$$

Multiply numerator and denominator by $(1 - \sin x)$.

$$= \frac{1 - \sin x}{1 - \sin^2 x}$$

Multiply.

Example 7 – *Solution*

cont'd

$$= \frac{1 - \sin x}{\cos^2 x}$$ Pythagorean identity

$$= \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}$$ Write as separate fractions.

$$= \frac{1}{\cos^2 x} - \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$ Product of fractions

$$= \sec^2 x - \tan x \sec x$$ Reciprocal and quotient identities

Verifying Trigonometric Identities

Guidelines for Verifying Trigonometric Identities

1. Work with one side of the equation at a time. It is often better to work with the more complicated side first.

2. Look for opportunities to factor an expression, add fractions, square a binomial, or create a monomial denominator.

3. Look for opportunities to use the fundamental identities. Note which functions are in the final expression you want. Sines and cosines pair up well, as do secants and tangents, and cosecants and cotangents.

4. If the preceding guidelines do not help, try converting all terms to sines and cosines.

5. Always try *something*. Even paths that lead to dead ends provide insights.

Example 1 – *Verifying a Trigonometric Identity*

Verify the identity $(\sec^2\theta - 1)/(\sec^2\theta) = \sin^2\theta$.

Solution:

$$\frac{\sec^2\theta - 1}{\sec^2\theta} = \frac{(\tan^2\theta + 1) - 1}{\sec^2\theta}$$ Pythagorean identity

$$= \frac{\tan^2\theta}{\sec^2\theta}$$ Simplify.

$$= \tan^2\theta(\cos^2\theta)$$ Reciprocal identity

$$= \frac{\sin^2\theta}{(\cos^2\theta)}(\cos^2\theta)$$ Quotient identity

$$= \sin^2\theta$$