We conclude this section with a summary of the differentiation rules studied so far. To become skilled at differentiation, you should memorize each rule.

Summary of Differentiation Rules

General Differentiation Rules

Let f, g, and u be differentiable functions of x.

Constant Multiple Rule:

$$\frac{d}{dx}[cf] = cf'$$

Sum or Difference Rule:

$$\frac{d}{dx}[f \pm g] = f' \pm g'$$

Product Rule:

$$\frac{d}{dx}[fg] = fg' + gf'$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$$

Derivatives of Algebraic Functions

Constant Rule:

$$\frac{d}{dx}[c] = 0$$

(Simple) Power Rule:

$$\frac{d}{dx}[x^n] = nx^{n-1}, \quad \frac{d}{dx}[x] = 1$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

Chain Rule

Chain Rule:

$$\frac{d}{dx}[f(u)] = f'(u)\,u'$$

General Power Rule:

$$\frac{d}{dx}[u^n] = nu^{n-1}\,u'$$

STUDY TIP As an aid to memorization, note that the cofunctions (cosine, cotangent, and cosecant) require a negative sign as part of their derivatives.

EXERCISES FOR SECTION 2.4

In Exercises 1–6, complete the table using Example 2 as a model.

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1. $y = (6x - 5)^4$		
2. $y = \dfrac{1}{\sqrt{x+1}}$		
3. $y = \sqrt{x^2 - 1}$		
4. $y = 3\tan(\pi x^2)$		
5. $y = \csc^3 x$		
6. $y = \cos \dfrac{3x}{2}$		

In Exercises 7–34, find the derivative of the function.

7. $y = (2x - 7)^3$

8. $y = (2x^3 + 1)^2$

9. $g(x) = 3(4 - 9x)^4$

10. $y = 3(4 - x^2)^5$

11. $f(x) = (9 - x^2)^{2/3}$

12. $f(t) = (9t + 2)^{2/3}$

13. $f(t) = \sqrt{1 - t}$

14. $g(x) = \sqrt{5 - 3x}$

15. $y = \sqrt[3]{9x^2 + 4}$

16. $g(x) = \sqrt{x^2 - 2x + 1}$

17. $y = 2\sqrt[4]{4 - x^2}$

18. $f(x) = -3\sqrt[4]{2 - 9x}$

Chain

19. $y = \dfrac{1}{x - 2}$

20. $s(t) = \dfrac{1}{t^2 + 3t - 1}$

21. $f(t) = \left(\dfrac{1}{t - 3}\right)^2$

22. $y = -\dfrac{5}{(t + 3)^3}$

23. $y = \dfrac{1}{\sqrt{x + 2}}$

24. $g(t) = \sqrt{\dfrac{1}{t^2 - 2}}$

25. $f(x) = x^2(x - 2)^4$

26. $f(x) = x(3x - 9)^3$

27. $y = x\sqrt{1 - x^2}$

28. $y = \frac{1}{2}x^2\sqrt{16 - x^2}$

29. $y = \dfrac{x}{\sqrt{x^2 + 1}}$

30. $y = \dfrac{x}{\sqrt{x^4 + 4}}$

31. $g(x) = \left(\dfrac{x + 5}{x^2 + 2}\right)^2$

32. $h(t) = \left(\dfrac{t^2}{t^3 + 2}\right)^2$

33. $f(v) = \left(\dfrac{1 - 2v}{1 + v}\right)^3$

34. $g(x) = \left(\dfrac{3x^2 - 2}{2x + 3}\right)^3$

In Exercises 35–44, use a computer algebra system to find the derivative of the function. Then use the utility to graph the function and its derivative on the same set of coordinate axes. Describe the behavior of the function that corresponds to any zeros of the graph of the derivative.

35. $y = \dfrac{\sqrt{x}+1}{x^2+1}$

36. $y = \sqrt{\dfrac{2x}{x+1}}$

37. $g(t) = \dfrac{3t^2}{\sqrt{t^2+2t-1}}$

38. $f(x) = \sqrt{x}(2-x)^2$

39. $y = \sqrt{\dfrac{x+1}{x}}$

40. $y = (t^2-9)\sqrt{t+2}$

41. $s(t) = \dfrac{-2(2-t)\sqrt{1+t}}{3}$

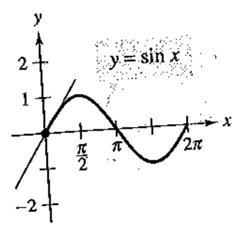
42. $g(x) = \sqrt{x-1} + \sqrt{x+1}$

43. $y = \dfrac{\cos \pi x + 1}{x}$

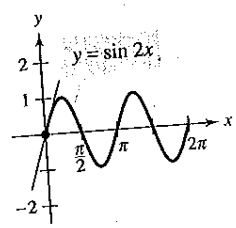
44. $y = x^2 \tan \dfrac{1}{x}$

In Exercises 45 and 46, find the slope of the tangent line to the sine function at the origin. Compare this value with the number of complete cycles in the interval $[0, 2\pi]$. What can you conclude about the slope of the sine function $\sin ax$ at the origin?

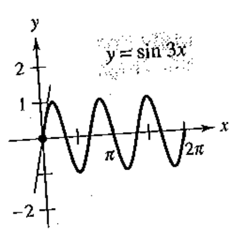
45. (a)



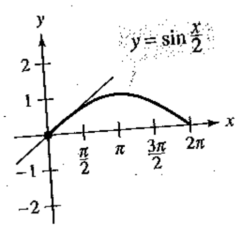
(b)



46. (a)



(b)



In Exercises 47–66, find the derivative of the function.

47. $y = \cos 3x$

48. $y = \sin \pi x$

49. $g(x) = 3 \tan 4x$

50. $h(x) = \sec x^2$

51. $y = \sin(\pi x)^2$

52. $y = \cos(1-2x)^2$

53. $h(x) = \sin 2x \cos 2x$

54. $g(\theta) = \sec(\tfrac{1}{2}\theta)\tan(\tfrac{1}{2}\theta)$

55. $f(x) = \dfrac{\cot x}{\sin x}$

56. $g(v) = \dfrac{\cos v}{\csc v}$

57. $y = 4 \sec^2 x$

58. $y = 2 \tan^3 x$

59. $f(\theta) = \tfrac{1}{4}\sin^2 2\theta$

60. $g(t) = 5 \cos^2 \pi t$

61. $f(t) = 3 \sec^2(\pi t - 1)$

62. $h(t) = 2 \cot^2(\pi t + 2)$

63. $y = \sqrt{x} + \tfrac{1}{4}\sin(2x)^2$

64. $y = 3x - 5\cos(\pi x)^2$

65. $y = \sin(\cos x)$

66. $y = \sin \sqrt[3]{x} + \sqrt[3]{\sin x}$

In Exercises 67–74, evaluate the derivative of the function at the indicated point. Use a graphing utility to verify your result.

Function	*Point*
67. $s(t) = \sqrt{t^2+2t+8}$	$(2, 4)$
68. $y = \sqrt[5]{3x^3+4x}$	$(2, 2)$
69. $f(x) = \dfrac{3}{x^3-4}$	$\left(-1, -\dfrac{3}{5}\right)$
70. $f(x) = \dfrac{1}{(x^2-3x)^2}$	$\left(4, \dfrac{1}{16}\right)$
71. $f(t) = \dfrac{3t+2}{t-1}$	$(0, -2)$
72. $f(x) = \dfrac{x+1}{2x-3}$	$(2, 3)$
73. $y = 37 - \sec^3(2x)$	$(0, 36)$
74. $y = \dfrac{1}{x} + \sqrt{\cos x}$	$\left(\dfrac{\pi}{2}, \dfrac{2}{\pi}\right)$

In Exercises 75–78, (a) find an equation of the tangent line to the graph of f at the indicated point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.

Function	*Point*
75. $f(x) = \sqrt{3x^2-2}$	$(3, 5)$
76. $f(x) = \tfrac{1}{3}x\sqrt{x^2+5}$	$(2, 2)$
77. $f(x) = \sin 2x$	$(\pi, 0)$
78. $f(x) = \tan^2 x$	$\left(\dfrac{\pi}{4}, 1\right)$

In Exercises 79–82, find the second derivative of the function.

79. $f(x) = 2(x^2-1)^3$

80. $f(x) = \dfrac{1}{x-2}$

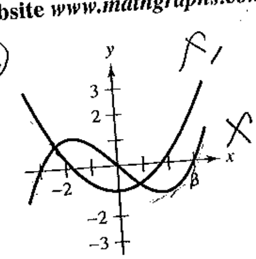
81. $f(x) = \sin x^2$

82. $f(x) = \sec^2 \pi x$

Getting at the Concept

In Exercises 83–86, the graphs of a function f and its derivative f' are shown. Label the graphs as f or f' and write a short paragraph stating the criteria used in making the selection. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

83.



84.